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An Innovative Numerical Comparison of Euler and Taylor Series Methods in Solving Differential Equations with Application to the Logistic Growth Equation

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Abstract

The objective of this work is to numerically compare the two numerical techniques for solving ordinary differential equations, namely the Euler method and the Taylor series method. The special focus of this numerical comparison will be on solving both the linear and nonlinear ordinary differential equations that can be modelled using the logistic growth model. The methodology employed in this study to achieve these objectives includes a detailed mathematical formulation of the problem followed by the numerical implementation of each method using the MATLAB computing environment and the determination of differences between the two methods with respect to their accuracy and computational efficiency. Additionally, a new hybrid method (the hybrid Euler-Taylor method), which uses a standard Euler approximation and applies a second order Taylor correction is also presented. This hybrid method provides an average performance between these two methods in terms of both accuracy and computational efficiency. The results show that while the Euler method is easy to implement, it is not very accurate, whereas the Taylor series method is very accurate but requires more computational effort. The hybrid Euler-Taylor method occupies the “sweet spot” between accuracy and computational efficiency.

Keywords: Mathematical Modeling, Approximate Solutions, Numerical Methods, Hybrid Euler–Taylor Method, Logistic Growth Model.

1- Introduction

Time-dependent changes in dynamic systems have commonly been represented by our most basic mathematical tools, which can be used to achieve the goal of developing models that have proven successful for a variety of applications in the social sciences, as well as the structural sciences and more.(Ali, 2025; Ascher, 2008) The mathematical techniques associated with differential equations form a basis for developing many types of computational models; examples include population growth, epidemiological studies, etc.,(Atkinson et al., 2009) with models that may be either linear or nonlinear depending on the nature of the underlying phenomenon. The ability to accurately and efficiently predict based on differential equations is critical when modeling nonlinear systems, necessitating numerical solutions techniques that will provide both an accurate and efficient means of determining approximate solutions to a variety of computational problems .(Butcher, 2021; Golub & Ortega, 2014) While the Euler formular can be used easily, its inaccuracy makes it an unsuitable way of approximating non-linear equations. (Gupta et al., 2024; Slavova, 2024) Instead, one might consider the Taylor series method. The Taylor series method combines multiple higher-order derivatives to form a sequential representation of the solution and produces a more accurate numerical approximation of the solution than does the Euler method;(Henner et al., 2023) thus, it will typically require more computing resources to perform than does the Euler method and the calculation of higher-order derivatives may be difficult .(Jumaeva, 2024).There have been several previous investigations addressing both the analysis and evaluation of the numerical performance of methods using approximate techniques for solving ordinary differential equations (for example, comparing Euler's Method with the Runge-Kutta method). These investigations have included evaluating how the Taylor Series can be used to increase the accuracy of a numerical solution to an ordinary differential equation; however, there are few investigations that have been completed from one perspective (generally with an emphasis on a target accuracy or being purely an analysis). Additionally, most of these investigations include only one analysis of numerical error, and no effort has been made to combine the error, numerical solution, and computation accuracies into one common structure. This is particularly important with respect to nonlinear models having complex behavior (for example, the logistic growth equation) . (Kiusalaas, 2010; Pelagalli et al., 2025)

The purpose of this paper is to provide a numerical comparison between the Euler and Taylor series methods in solving linear and non-linear equations. The research will use the Logistic Growth Model to illustrate non-linear behavior and introduce an innovative hybrid numerical technique that benefits from both the ease of using the Euler method and the accuracy of using the Taylor series method. In developing this new technique, corrections will be applied from the Taylor series expansion to the Euler method to reduce errors and improve accuracy while maintaining computational simplicity. What the research will provide to the world will be a much larger body of information than is available from the typical two-methods-traditional way of doing things. More importantly, it will provide an analytical framework for integrating or combining both numerical and arithmetic methodologies of accuracy; as well as allow for testing of the proposed hybrid method of solution to determine its performance via empirical application of the successive improvement proposed operationally on an actual linear and/or nonlinear system application (e.g., logistic growth equation). Furthermore, it will also represent a highly significant contribution to science, in that no other research study has forged a bridge across the five major divides in scientific discipline (i.e., combination of numeric and non-numeric methodologies in one application model), and thus will be an important tool to help researchers select an appropriate numeric methodology to use for each quantitative question they pose by being able to apply the correct method of numerical analysis (no two questions can be answered with the same numeric methodology).

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2-Euler's Method

It is one of the simplest and most popular numerical methods used to solve ordinary differential equations. It is named after the famous mathematician Leonard Euler.(Racshitha et al., 2025) This method is based on the principle of repetition, where the slope (derivative) at a certain point is used to calculate the next point. In doing so, Euler's method approximates the solution step by step by dividing the time period into small parts and calculating the change in each part. The basic mathematical formula of the Euler method (Salgado & Romanelli, 2024)

$\mathcal{Y}_{\eta+1} = \mathcal{Y}_{\eta} + \hbar . f(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta})$

Whereas:

$\mathcal{Y}_{\eta}$ Represents the present value of the solution.

$\mathcal{Y}_{\eta+1}$ The following value represents the solution.

$\hbar$ Represents the size of the step (the distance between points on the $\mathcal{X}$ -axis).

$f(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta})$ represents the slope (derived at the point $(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta})$)

$\mathcal{X}_{\eta} = \mathcal{X}_0 + \eta . \hbar$ Whereas: $\mathcal{X}_0$ Represents the initial value ، η Represents the number of repetitions.

Example 1:

To find the approximate solution of the initial condition problem $\mathcal{Y}(0) = 1$ and $\mathcal{Y}' = -\mathcal{Y} + \mathcal{X} + 1$where $\hbar = 0.1$and $0 < \mathcal{X} < 1$ and compare to the correct solution $\mathcal{Y} = \mathcal{X} + e^{-\mathcal{X}}$.

Solution: To find an approximate solution using Euler's formula:

$$\mathcal{Y}_{\eta+1} = \mathcal{Y}_{\eta} + \hbar . f(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta})$$

$$F(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) = -\mathcal{Y}_{\eta} + \mathcal{X}_{\eta} + 1$$

We will do the following steps:

$$\mathcal{Y}_{\eta+1} = \mathcal{Y}_{\eta} + \hbar(-\mathcal{Y}_{\eta} + \mathcal{X}_{\eta} + 1)$$

$\mathcal{Y}_0 = 1$, $\mathcal{X}_0 = 0$

Let's get the values as shown in the table (1):

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Table 1: Comparison of the Right Solution and the Approximate Solution Using the Euler Method

η	$\mathcal{X}_\eta$	Numerical solution	Exact solution	Error
0	0	1	1	0.0000000
1	0.1	1	1.004837	4.837×10^{-3}
2	0.2	1.01	1.018731	8.731×10^{-3}
3	0.3	1.029	1.040818	0.011818
4	0.4	1.0561	1.07032	0.01422
5	0.5	1.09049	1.106531	0.016041
6	0.6	1.131441	1.148812	0.17371
7	0.7	1.178297	1.196585	0.182883
8	0.8	1.230467	1.249329	0.018862
9	0.9	1.28742	1.30657	0.019201
10	1	1.348678	1.367879	0.019201

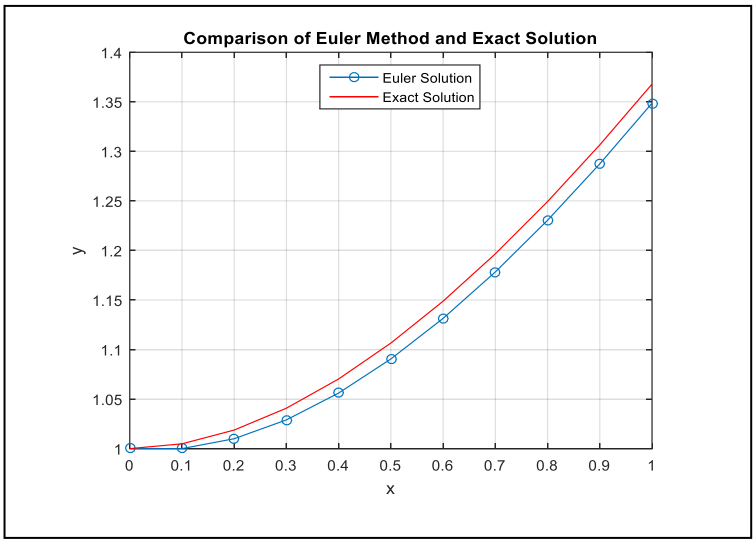

Fig 1: Shows a comparison between the correct and approximate solution using the Euler method

Example 2:

To find an approximate solution to the problem of the initial condition of logistic growth:

$\mathcal{Y}' = r\mathcal{Y}\left(1-\frac{\mathcal{Y}}{k}\right)$, $\mathcal{Y}(0) = 1$, where $\hbar = 0.1$, $r = 0.5$, $k = 10,\ 0 < \mathcal{X} < 1$ and compare it to the correct solution $\mathcal{Y} = \frac{10}{1+9e^{-0.5\mathcal{X}}}$ Using the Euler method:

Solution: To find an approximate solution using Euler's formula:

$\mathcal{Y}_{\eta+1} = \mathcal{Y}_\eta + \hbar . f(\mathcal{X}_\eta, \mathcal{Y}_\eta)$

$$F(\mathcal{X}_\eta, \mathcal{Y}_\eta) = r\mathcal{Y}_\eta\left(1 - \frac{\mathcal{Y}_\eta}{k}\right)$$

We will do the following steps : $\mathcal{Y}_{\eta+1} = \mathcal{Y}_\eta + \hbar\left(r\mathcal{Y}_\eta\left(1 - \frac{\mathcal{Y}_\eta}{k}\right)\right)$, $\mathcal{Y}_0 = 1$, $\mathcal{X}_0 = 0$

Let's get the values as shown in the table(2):

Table 2: Comparison of the Exact Solution and the Approximate Solution Using the Euler Method

η	$\mathcal{X}_\eta$	**Numerical solution**	**Exact solution**	**Error**
0	0.0	1.0000	1.0000	0.0000
1	0.1	1.0450	1.0459	0.0009
2	0.2	1.0918	1.0937	0.0019
3	0.3	1.1404	1.1433	0.0029
4	0.4	1.1909	1.1949	0.0040
5	0.5	1.2434	1.2486	0.0052
6	0.6	1.2978	1.3042	0.0064
7	0.7	1.3543	1.3620	0.0077
8	0.8	1.4128	1.4219	0.0090
9	0.9	1.4735	1.4840	0.0105
10	1	1.5363	1.5483	0.0120

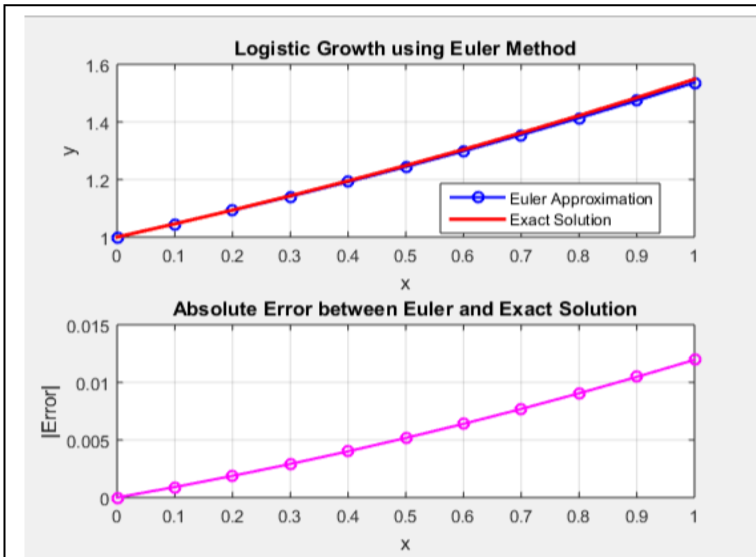

Fig 2: Shows a comparison between the correct and approximate solution using the Euler method and the amount of absolute error at each point

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By comparing the numerical solution of the Euler method with the analytical solution of the logistic growth problem, we can see that the Euler method is able to give a good approximation at the small values of the variable, but its accuracy gradually decreases as the value of $\mathcal{X}$ increases as the accumulation of error in each step. It is also clear that the size of step $\hbar$ plays a key role in reducing this error, as choosing a smaller value for h leads to more accurate results at the expense of increasing the number of calculations.

3-Taylor series method

An important mathematical method for solving ordinary differential equations, this method is based on representing the solution as a series, which is the representation of a function as an infinite sum of a number of terms based on its derivatives at a given point.(Sauer, 2025) This method is used in cases where the details of the function can be obtained mathematically, and it is distinguished from some other methods by not being affected by the rotation error (which is the error resulting from the use of a specific number of significant ranks), just as the differences used in the mentioned methods are affected and the value of h can be used rather large. To find the solution to the equation, $\frac{d\mathcal{Y}}{d\mathcal{X}} = f(\mathcal{X}, \mathcal{Y})$, provided that the solution passes the point $(\mathcal{X}_0, \mathcal{Y}_0)$,which represents initial condition that makes it difficult to integrate the equation (to find its solution), the formula of the Tyler series is formed: (Salgado & Romanelli, 2024; Tumuluri, 2021)

$$\mathcal{Y}(\mathcal{X}) = \mathcal{Y}(\mathcal{X}_0) + (\mathcal{X} - \mathcal{X}_0)\mathcal{Y}(\mathcal{X}_0)' + \frac{(\mathcal{X} - \mathcal{X}_0)^2}{2!}\mathcal{Y}(\mathcal{X}_0)'' + \frac{(\mathcal{X} - \mathcal{X}_0)^3}{3!}\mathcal{Y}(\mathcal{X}_0)''' + \cdots \frac{(\mathcal{X} - \mathcal{X}_0)^{\eta}}{\eta!}\mathcal{Y}(\mathcal{X}_0)^{\eta} \qquad \ldots.(1)$$

If we have a function $y(\mathcal{X})$ Sufficiently reducible the Tyler series is around the point $\mathcal{X} = \mathcal{X}_0$ In terms of y and $\hbar$ where $\hbar = \mathcal{X} - \mathcal{X}_0$

$$\mathcal{Y}(\mathcal{X}) = \mathcal{Y}(\mathcal{X}_0) + \hbar\mathcal{Y}(\mathcal{X}_0)' + \frac{\hbar^2}{2!}\mathcal{Y}(\mathcal{X}_0)'' + \frac{\hbar^3}{3!}\mathcal{Y}(\mathcal{X}_0)''' + \ \ldots\frac{\hbar^{\eta}}{\eta!}\mathcal{Y}(\mathcal{X}_0)^{\eta} \qquad \ldots.(2)$$

Relation (2) of the η -order Tyler chain can be generalized as follows:

$\mathcal{Y}_{\eta+1} = \mathcal{Y}_{\eta} + \hbar.T^{\eta}(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta})$ for each η =0,1,2, ...

Whereas: $T(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) = [f(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) + \frac{\hbar}{2}f'(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) + \frac{\hbar^2}{3!}f''(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) + \frac{\hbar^3}{4!}f'''(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta})]$

Example 3:

To find the approximate solution of example (1) and compare it to the correct solution using the Tyler String Method of the fourth order, we will do the following steps:

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$$\mathcal{Y}_{\eta+1} = \mathcal{Y}_{\eta} + \hbar . T^4(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta})$$

Where: $f(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) = -\mathcal{Y}_{\eta} + \mathcal{X}_{\eta} + 1$, $f'(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) = \mathcal{Y}_{\eta} - \mathcal{X}_{\eta}$

$f''(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) = \mathcal{Y}_{\eta} + \mathcal{X}_{\eta}$, $f'''(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) = \mathcal{Y}_{\eta} - \mathcal{X}_{\eta}$

By Relationship Compensation:

$$T^4\, F(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) = [(-\mathcal{Y}_{\eta} + \mathcal{X}_{\eta} + 1) + \frac{\hbar}{2!}(\mathcal{Y}_{\eta} - \mathcal{X}_{\eta}) + \frac{\hbar^2}{3!}(\mathcal{Y}_{\eta} - \mathcal{X}_{\eta}) + \frac{\hbar^3}{4!}(\mathcal{Y}_{\eta} - \mathcal{X}_{\eta})]$$

$$\mathcal{Y}_1 = \mathcal{Y}_0 + \hbar T^4(\mathcal{X}_0, \mathcal{Y}_0)$$

This is how we can get the values and the table (3) represents the resulting values, and the values of the correct solution are the same as we got in Example (1).

Table 3: Comparison of the Right Solution and the Approximate Solution Using the Tyler String Method of the Fourth Order

η	$\mathcal{X}_{\eta}$	**Numerical solution**	**Exact solution**	**Error**
0	0	1	1	0.00000
1	0.1	1.0048375	1.004837	5×10^{-7}
2	0.2	1.018730901	1.018731	9.9×10^{-8}
3	0.3	1.04081842	1.040818	4.22×10^{-7}
4	0.4	1.070320289	1.070329	2.89×10^{-7}
5	0.5	1.106530934	1.106531	6.6×10^{-7}
6	0.6	1.148811934	1.148812	6.6×10^{-7}
7	0.7	1.196585618	1.1965853	3.18×10^{-7}
8	0.8	1.249329289	1.249329289	2.89×10^{-7}
9	0.9	1.3065699	1.330656991	9×10^{-7}
10	1	1.3678797	1.367879	7.74×10^{-7}

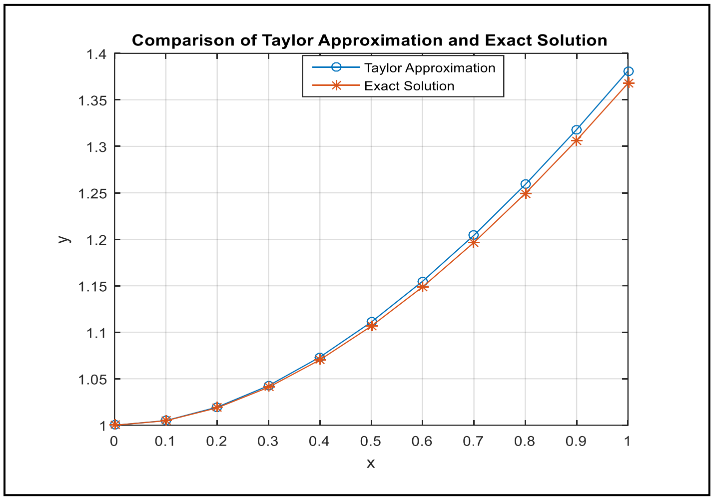

Fig 3: Shows a comparison between the correct and approximate solution using the Tyler series method

Example 4:

To find the approximate solution of example (2) and compare it to the correct solution using the Tyler String Method of the fourth order, we will do the following steps:

1. Logistic growth equation:

$$\mathcal{Y} = r\mathcal{Y}\left(1 - \frac{\mathcal{Y}}{k}\right), \mathcal{Y}(0) = 1$$

2. Fourth-order Taylor method:

$$\mathcal{Y}_{\eta+1} = \mathcal{Y}_{\eta} + \hbar\mathcal{Y}'_{\eta} + \frac{\hbar^2}{2!}\mathcal{Y}"_{\eta} + \frac{\hbar^3}{3!}\mathcal{Y}'''_{\eta} + \frac{\hbar^4}{4!}\mathcal{Y}^{(4)}_{\eta}$$

3. Time derivatives:

$$\mathcal{Y}'_{\eta} = f(\mathcal{Y}_{\eta}) \quad , \quad \mathcal{Y}"_{\eta} = f'(\mathcal{Y}_{\eta})f(\mathcal{Y}_{\eta})$$
$$\mathcal{Y}'''_{\eta} = f''(\mathcal{Y}_{\eta})f(\mathcal{Y}_{\eta})^2 + (f'(\mathcal{Y}_{\eta}))^2 f(\mathcal{Y}_{\eta})$$
$$\mathcal{Y}^{(4)}_{\eta} = (4f''(\mathcal{Y}_{\eta})f(\mathcal{Y}_{\eta})f'(\mathcal{Y}_{\eta}) + \left(f'(\mathcal{Y}_{\eta})\right)^3 f(\mathcal{Y}_{\eta})$$

4. Derivatives with respect to y:

$$f(\mathcal{Y}) = r\mathcal{Y}\left(1 - \frac{\mathcal{Y}}{k}\right) \quad , \quad f'(\mathcal{Y}) = r\left(1 - \frac{2\mathcal{Y}}{k}\right) \quad , \quad f''(\mathcal{Y}) = -\frac{2r}{k}$$

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5. Final update formula:

$$\mathcal{Y}_{\eta+1} = \mathcal{Y}_\eta + \hbar f + \frac{\hbar^2}{2}(f'(\mathcal{Y}_\eta)f) + \frac{\hbar^3}{3}\left(f''(\mathcal{Y}_\eta)f^2 + \left(f'(\mathcal{Y}_\eta)\right)^2 f\right) + \frac{\hbar^4}{4}\left(\left(4f''(\mathcal{Y}_\eta)f^2 f'(\mathcal{Y}_\eta) + \left(f'(\mathcal{Y}_\eta)\right)^3\right) f\right)$$

by compensating for the following values:
$\mathcal{Y}_0 = 1$, $\mathcal{X}_0 = 0$, $\hbar = 0.1$, $r = 0.5$, $k = 10$, $0 < \mathcal{X} < 1$

In this way, we repeat to calculate the rest of the points in the same way, The following table (4) shows us the rest of the values:

Table 4: Comparison of the Right Solution and the Approximate Solution Using the Tyler String Method of the Fourth Order

$\mathcal{X}$	$\mathcal{Y}$ _Taylor4	$\mathcal{Y}$ _exact	Absolute Error
0.0	1.000000	1.000000	0.00×10^{0}
0.1	1.045909	1.045909	1.73×10^{-9}
0.2	1.093669	1.093669	3.67×10^{-9}
0.3	1.143331	1.143331	5.83×10^{-9}
0.4	1.194946	1.194946	8.21×10^{-9}
0.5	1.248563	1.248563	1.08×10^{-8}
0.6	1.304229	1.304229	1.37×10^{-8}
0.7	1.361991	1.361991	1.67×10^{-8}
0.8	1.421893	1.421893	2.00×10^{-8}
0.9	1.483976	1.483976	2.35×10^{-8}
1.0	1.548281	1.548281	2.73×10^{-8}

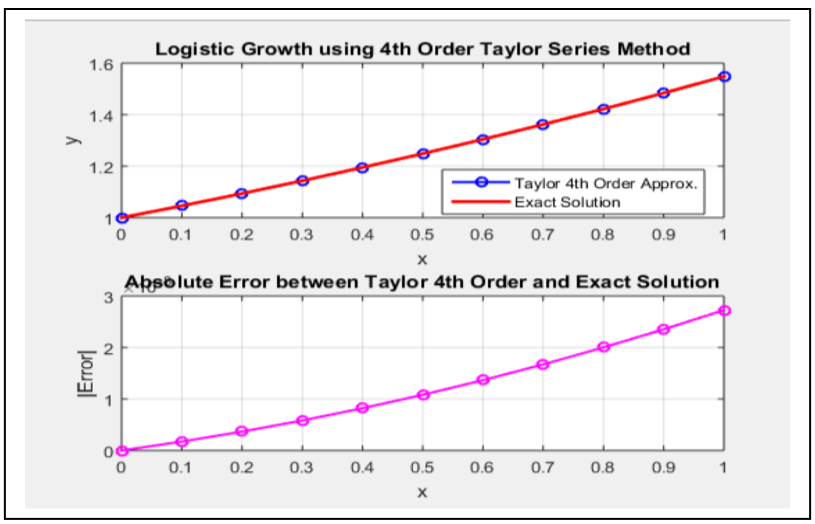

Fig 4: Shows a comparison between the correct and approximate solution using the Tyler series method and the amount of absolute error at each point.

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Through the numerical experiment, it was found that the fourth-order Taylor method gave results that are almost identical to the analytical solution of the logistic growth equation under the given values (r=0.5,k=10) and the initial condition ($\mathcal{Y}(0)$=1).
The absolute and relative errors within the period 0< $\mathcal{X}$ ≤1 were very small (with a rank of $10^{-9}, 10^{-8}$), demonstrating the high accuracy of this method even with the choice of a medium-sized step $\hbar$ =0.1. The Taylor method is a reliable and effective tool for solving nonlinear growth models such as the logistic equation, and it significantly outperforms the Euler method in terms of accuracy while maintaining computational efficiency.

4-Euler-Taylor Hybrid Method

The Euler–Taylor hybrid method is an innovative numerical method that combines the Euler method and the Taylor series. This method develops the Euler method by introducing a second-order corrective limit of the Taylor series, which contributes to reducing error without a significant increase in computational complexity. the formula of the method is as follows:

$$\mathcal{Y}_{\eta+1} = \mathcal{Y}_{\eta} + \hbar . f\left(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}\right) + \omega \frac{\hbar^2}{2} f'(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta})$$

Where (ω) represents the effect coefficient of the correction limit of the Taylor series. This hybrid formula provides a proper balance between accuracy and efficiency, making it suitable for solving linear and nonlinear differential equations, including the logistic growth model.

Example 5:

To find the approximate solution of example (1) and compare it to the correct solution using the Euler-Taylor Hybrid Method ، when this is considered $\omega = 1$ a correction of the Taylor series in high resolution, we will do the following steps:

$$\mathcal{Y}_{\eta+1} = \mathcal{Y}_{\eta} + \hbar . f\left(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}\right) + \omega \frac{\hbar^2}{2} f'(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta})$$

Where: $f(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) = -\mathcal{Y}_{\eta} + \mathcal{X}_{\eta} + 1$, $f'(\mathcal{X}_{\eta}, \mathcal{Y}_{\eta}) = \mathcal{Y}_{\eta} - \mathcal{X}_{\eta}$

$$\mathcal{Y}_{\eta+1} = \mathcal{Y}_{\eta} + \hbar(-\mathcal{Y}_{\eta} + \mathcal{X}_{\eta} + 1) + \omega \frac{\hbar^2}{2} (\mathcal{Y}_{\eta} - \mathcal{X}_{\eta})$$

This is how we can get the values and the table (5) represents the resulting values, and the values of the correct solution are the same as we got in Example (1).

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Table 5: Comparison of the Right Solution and the Approximate Solution Using the Euler–Taylor hybrid method

η	x_η	Numerical solution	Exact solution	Error
0	0	1	1	0.00000
1	0.1	1.005000	1.004837	0.000163
2	0.2	1.019025	1.018731	0.000295
3	0.3	1.041218	1.040818	0.000400
4	0.4	1.070647	1.070329	0.000327
5	0.5	1.106348	1.106531	0.000183
6	0.6	1.147410	1.148812	0.000095
7	0.7	1.192982	1.1965853	0.000010
8	0.8	1.242282	1.249329289	0.000129
9	0.9	1.294600	1.330656991	0.000234
10	1	1.349967	1.367879	0.017912

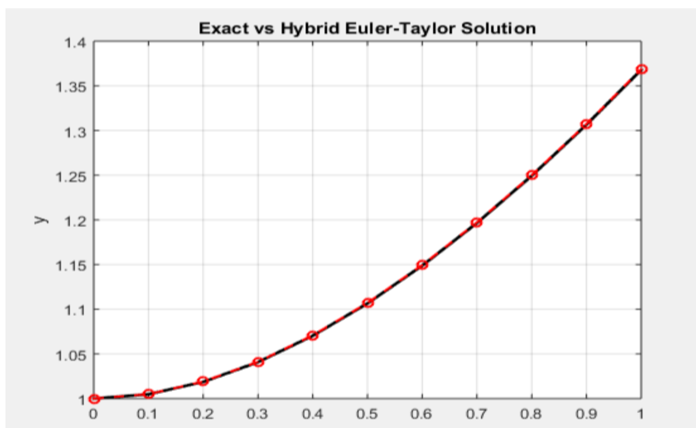

Fig 5: Shows a comparison between the correct and approximate solution using The Euler–Taylor hybrid method

We notice the error is very small at all points x and the hybrid method gave high accuracy, and results very close to the real solution.

Example 6:

To find the approximate solution of example (2) and compare it to the correct solution using the Euler–Taylor hybrid method, we will do the following steps:

Solution: To find an approximate solution using Euler–Taylor hybrid formula:

1. $\mathcal{Y}_{\eta+1} = \mathcal{Y}_\eta + \hbar . f(\mathcal{X}_\eta, \mathcal{Y}_\eta) + \omega \frac{\hbar^2}{2} f'(\mathcal{X}_\eta, \mathcal{Y}_\eta) \quad , \omega = 1$
2. Logistic growth equation:

$$F(\mathcal{X}, \mathcal{Y}) = 0.5\mathcal{Y}\left(1 - \frac{\mathcal{Y}}{10}\right) \;, \mathcal{Y}(0) = 1 \,, r = 0.5 \,, \hbar = 0.1$$

$$F'(\mathcal{X},\mathcal{Y}) = 0.5\mathcal{Y}\left(1 - \frac{2\mathcal{Y}}{10}\right)$$

Let's get the values as shown in the table (6) :

Table 6: Comparison of the Right Solution and the Approximate Solution Using the Euler–Taylor hybrid method

η	$\mathcal{X}_η$	Numerical solution	Exact solution	Error
0	0	1.0	1.0	0.0
1	0.1	1.046125	1.046054	0.000071
2	0.2	1.094553	1.09443	0.000123
3	0.3	1.145343	1.145142	0.000201
4	0.4	1.198552	1.198205	0.000347
5	0.5	1.254232	1.253951	0.000281
6	0.6	1.31243	1.312119	0.000311
7	0.7	1.373186	1.372851	0.000335
8	0.8	1.436533	1.43618	0.000353
9	0.9	1.502496	1.502129	0.000367
10	1	1.571093	1.570704	0.000389

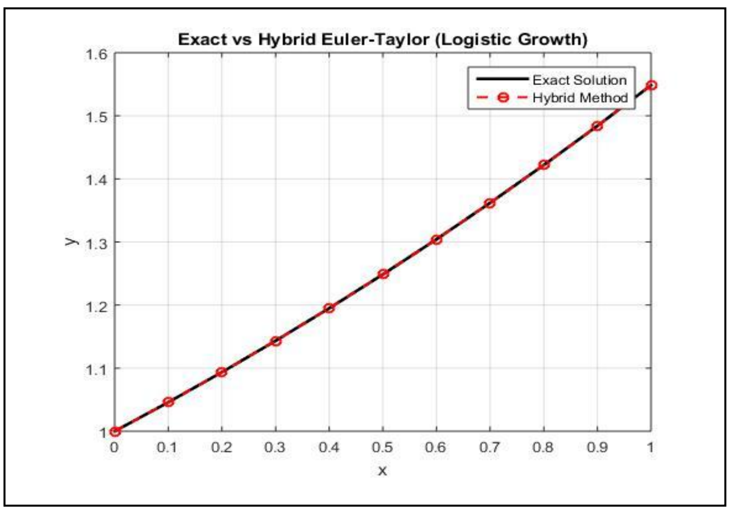

Fig **6**: Shows a comparison between the correct and approximate solution using The Euler–Taylor hybrid method

We note that the error is very small and the hybrid method gives excellent affinity with the real solution and the numerical behavior follows the logistic growth accurately.

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5-Comparison of the Euler, Tyler Series and Euler–Taylor hybrid methods

We will show the results obtained using the Euler, Tyler Series and Euler–Taylor hybrid methods with the correct solution for the ideals (1), (3) and (5) in Table (7):

Table7: Comparison of the correct solution and the approximate solution using the Euler, Tyler Series and Euler–Taylor hybrid methods

ƞ	$x_ƞ$	Exact solution	Euler	Tyler	Euler–Taylor hybrid
0	0	1	1	1	1
1	0.1	1.004837	1	1.0048375	1.004837
2	0.2	1.018731	1.01	1.0189730	1.018731
3	0.3	1.040818	1.029	1.0408184	1.040818
4	0.4	1.07032	1.0561	1.0703202	1.070329
5	0.5	1.106531	1.09049	1.1065329	1.106531
6	0.6	1.148812	1.131441	1.148812	1.148812
7	0.7	1.196585	1.78297	1.1965856	1.1965853
8	0.8	1.249329	1.230467	1.293292	1.249329289
9	0.9	1.30657	1.28742	1.3065699	1.330656991
10	1	1.367879	1.348678	1.3678797	1.367879

We will show the errors resulting from the use of the three methods for ideals (1), (3) and (5) in Table (8):

Table 8: Comparison of the Error Amount Using the Euler, Tyler Series and Euler–Taylor hybrid Methods

ƞ	$x_ƞ$	Euler	Tyler	Euler–Taylor hybrid
0	0	0.0000000	0.00000	0.00000
1	0.1	4.837×10^{-3}	5×10^{-7}	0.000163
2	0.2	8.731×10^{-3}	9.9×10^{-8}	0.000295
3	0.3	0.011818	4.22×10^{-7}	0.000400
4	0.4	0.01422	2.89×10^{-7}	0.000327
5	0.5	0.016041	6.6×10^{-7}	0.000183
6	0.6	0.17371	6.6×10^{-7}	0.000095
7	0.7	0.182883	3.18×10^{-7}	0.000010
8	0.8	0.018862	2.89×10^{-7}	0.000129
9	0.9	0.019201	9×10^{-7}	0.000234
10	10	0.019201	7.74×10^{-7}	0.017912

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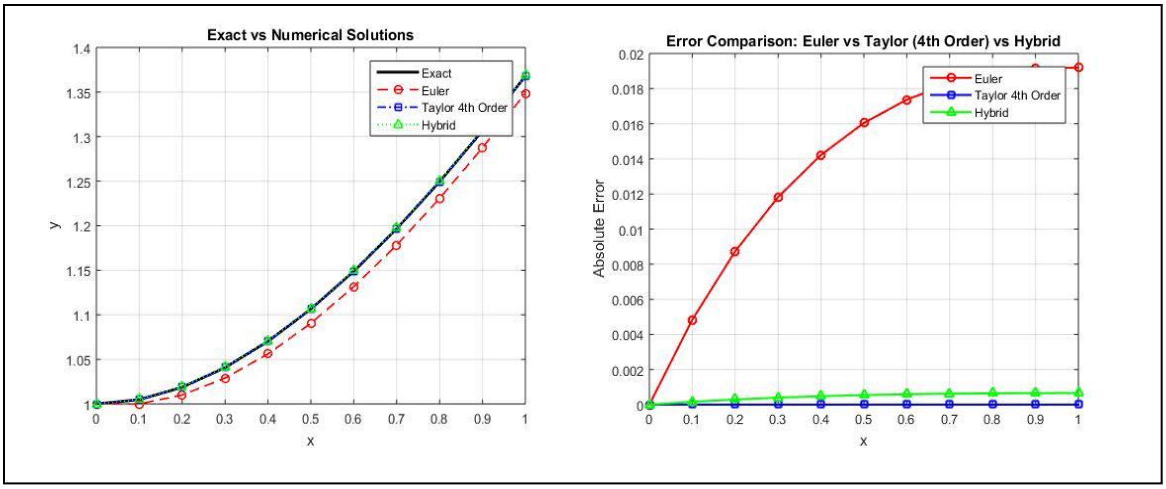

Fig **7**: showing the comparison of the real solution and the amount of error with the numerical solution using the three methods

We will show the results obtained using the Euler, Tyler Series and Euler–Taylor hybrid methods with the correct solution for the ideals (2), (4) and (6) in Table (9):

Table 9: Comparison of the correct solution and the approximate solution using the Euler, Tyler Series and Euler–Taylor hybrid methods

η	$x_η$	Exact solution	Euler	Tyler	Euler–Taylor hybrid
0	0	1.0	1.0000	1.000000	1.0
1	0.1	1.046054	1.0450	1.045909	1.046125
2	0.2	1.09443	1.0918	1.093669	1.094553
3	0.3	1.145142	1.1404	1.143331	1.145343
4	0.4	1.198205	1.1909	1.194946	1.198552
5	0.5	1.253951	1.2434	1.248563	1.254232
6	0.6	1.312119	1.2978	1.304229	1.31243
7	0.7	1.372851	1.3543	1.361991	1.373186
8	0.8	1.43618	1.4128	1.421893	1.436533
9	0.9	1.502129	1.4735	1.483976	1.502496
10	1	1.570704	1.5363	1.548281	1.571093

We will show the errors resulting from the use of the three methods for ideals (2), (4) and (6) in Table (10):

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Table 10: Comparison of the Error Amount Using the Euler, Tyler Series and Euler–Taylor hybrid Methods

η	x_η	Euler	Tyler	Euler–Taylor hybrid
0	0	0.0000	0.00	0.0
1	0.1	0.0009	1.73×10^{-9}	0.000071
2	0.2	0.0019	3.67×10^{-9}	0.000123
3	0.3	0.0029	5.83×10^{-9}	0.000201
4	0.4	0.0040	8.21×10^{-9}	0.000347
5	0.5	0.0052	1.08×10^{-8}	0.000281
6	0.6	0.0064	1.37×10^{-8}	0.000311
7	0.7	0.0077	1.67×10^{-8}	0.000335
8	0.8	0.0090	2.00×10^{-8}	0.000353
9	0.9	0.0105	2.35×10^{-8}	0.000367
10	10	0.0120	2.73×10^{-8}	0.000389

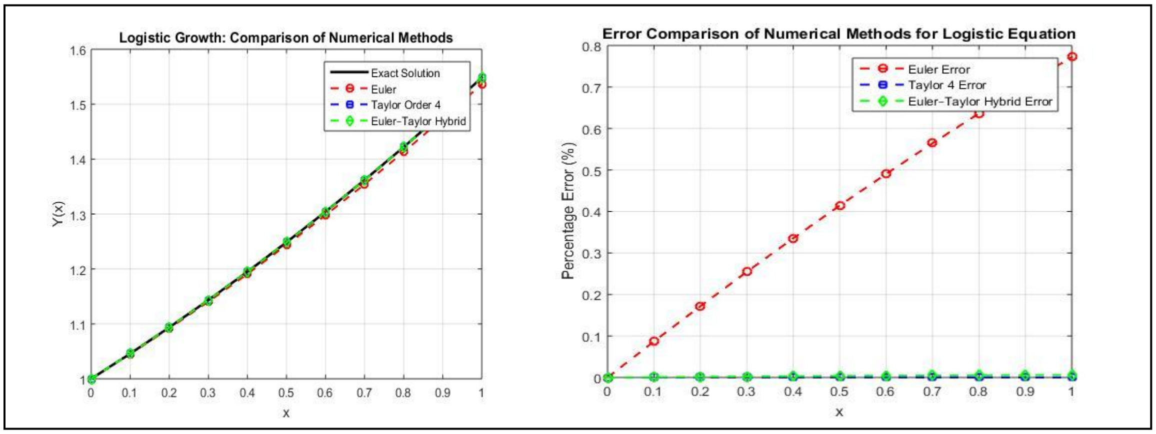

Fig 8: showing the comparison of the real solution and the amount of error with the numerical solution using the three methods

6-Conclusion

The numerical results in this research, showed that the Euler method represents an effective and simple method in approximating the solutions of differential equations, but its accuracy is clearly affected by the increase in the amount of error, especially when applied to nonlinear equations, such as the logistic growth equation. While the fourth-order Taylor series method achieved great accuracy in both linear and nonlinear equations, it did not rely on higher derivatives that reduced the error value and gave an approximation towards the real solution. Thus, the numerical comparison proved that the introduction of a hybrid method based on the correction of the Euler method using Taylor's expansion, which contributed to improving the numerical solution and reducing the error without computational complexity, which in turn enhanced the efficiency of numerical solutions in practical applications. The importance of this study, lies in highlighting a (innovative) numerical comparison by combining the analysis between more than one numerical

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method and evaluating it using the error value, this allows a deeper understanding of the behavior of numerical methods, when applied to linear and nonlinear differential equations, and confirms that the selection of the appropriate method depends on the balance between accuracy and computational efficiency.

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