

The Best Common Feature Between ii-Open and α –Open Sets

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Abstract:

It is well known that for any topological space the family of all α –open sets is also a topological space. In this paper, we introduce a new description of ii-open sets in a topological space with proof that the family of ii-open sets is again a topological space.

Keywords:

α –open sets and ii-open sets

1. Introduction and Preliminaries

A systematic study of ii-open sets in topological spaces are presented recently in [8], [4] and [5]. The semi-open [6] and α –open sets [9] are considered the start of the appearance of the new classes of open sets that they came after, see for example [1], [3], [8] and [2].

Let (X, τ) be a topological space. Recall that, $M \subseteq X$ is open, if $M \in \tau$ or for all $m \in M$ there exists $O \in \tau$ such that $m \in O \subseteq M$. Also, M is closed if M^c is open, M is α –open if $M \subseteq \text{int}(cl(\text{int}(M)))$, M is int-open [8] if there exists $\phi, X \neq$

$O \in \tau$ such that $\text{int}(M) = O$ and M is i -open [7] if there exists $\phi, X \neq O \in \tau$ such that $M \subseteq \text{cl}(M \cap O)$.

In this paper, we denoted by $\tau^\circ, \tau^c, \tau^i, \tau^{\text{int}}$ and τ^α to be the family of open, closed, i -open, int -open and α -open sets respectively. Finally, $M \subseteq X$ is said to be to be ii -open if $M \in \tau^i \cap \tau^{\text{int}}$ and we denote by τ^{ii} to be the family of all ii -open sets.

2. Proving (X, τ^{ii}) is a topological space.

We begin this section by the following theorem:

Theorem2.1. ([7], [8]) For any topological space, all open sets are int -open, i -open, ii -open and α -open. Further, all α -open and ii -open sets are i -open.

Note that the converse of above theorem is not true in general.

Example2.2. Let $X = \{0, 1, 2, \dots\}$,

$\tau = \{\phi, \{0, 2, 3, \dots\}, \{1, 2, 3, \dots\}, \{2, 3, 4, \dots\}, \{0\}, X\}$,

$M = \{0, 2\}$ is ii -open not α -open because $\text{int}(M) = \{0\}$, so $M \in \tau^{\text{int}}$.

Now, $M \subseteq \text{Cl}(M \cap \{0, 2, 3, \dots\})$, thus, $M \in \tau^i$ and therefore, $M \in \tau^{ii}$.

But $\text{Cl}(\text{int}(M)) = \{0\}$ and $M = \{0, 2\} \not\subseteq \text{int}(\text{Cl}(\text{int}(M)))$.

Example2.3. Let $X = \{0, 1, 2\}$ and $\tau_1 = \{\phi, \{0\}, X\}$, $\tau_2 = \{\phi, \{0\}, \{1, 2\}, X\}$, $\tau_3 = \{\phi, \{0\}, \{1\}, \{0, 1\}, \{0, 2\}, X\}$. Then, $\tau_1^\alpha = \tau_1^{\text{int}} = \tau_1^i = \tau_1^{ii} = \{\phi, \{0\}, \{0, 1\}, \{0, 2\}, X\}$ and $\tau_2^{\text{int}} = \{\phi, \{0\}, \{1, 2\}, \{0, 1\}, \{0, 2\}, X\}$, $\tau_2^i = \{\phi, \{0\}, \{1, 2\}, \{1\}, \{2\}, X\}$. Therefore, $\tau_2^{ii} = \tau_2^i \cap \tau_2^{\text{int}} = \{\phi, \{0\}, \{1, 2\}, X\}$ and $\tau_2^\alpha = \tau_2^{\text{int}} = \tau_2^{ii}$. Finally, $\tau_3^{\text{int}} = \{\phi, \{0\}, \{1\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, X\}$, $\tau_3^i = \{\phi, \{0\}, \{1\}, \{0, 1\}, \{0, 2\}, \{2\}, X\}$. Therefore, $\tau_3^{ii} = \tau_3$ and $\tau_3^\alpha = \tau_3^{ii}$.

Now, from Theorem 2.1 and above examples we conclude the following diagram:

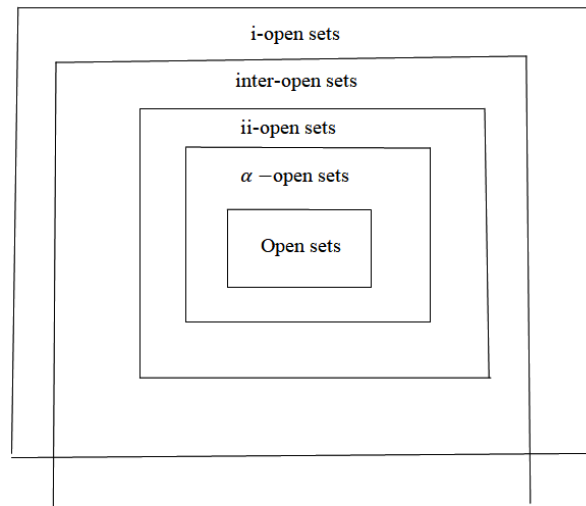


Fig.1.

Now, we prove our main result:

Theorem 2.4. (X, τ^{ii}) is a topological space.

Proof. Let $M \subseteq X$ be ii-open set. We will prove that M is open in (X, τ^{ii}) . That is, for all $m \in M$ there exists $O^* \in \tau^{ii}$ such that $m \in O^* \subseteq M$. Since M is ii-open set, it follows that there exists $\phi, X \neq O, K \in \tau$ such that $\text{int}(M) = O$ and $M \subseteq \text{Cl}(M \cap K)$. Now, for $m \in M$ we have the following two cases:

- (i) $m \in O$.
- (ii) $m \notin O$.

Case (i) $m \in O$. By Theorem 2.1, O is ii-open, so put $O = O^*$ we have, $m \in O^* = O = \text{int}(M) \subseteq M$. Thus, M is open in τ^{ii} . Case (ii) $m \notin O$. Consider the following sets, $O_o = \{m\} \cup O$ and $O_n = \{m\} \cup \{m_i\}_{i=1}^n \cup O, \forall n = 1, 2, \dots$ with $m_i \in M \forall i$. Put $O^* = O_n$ for some $n \in \{0, 1, 2, \dots\}$. Therefore, it is enough to see that O^* is ii-open. Clearly, $O^* \subseteq \text{Cl}(O^* \cap O)$ and $\text{int}(O^*) = O$. This implies, $m \in O^* \subseteq M$.

Now, since every set in τ^{ii} is open, it follows that τ^{ii} is closed under arbitrary union and finite intersection. This complete the proof.

Remark2.5. Let $M \subseteq X$, then M is ii-open if $M \subseteq Cl(M \cap O)$ and $int(M) = K$. If we take $O = K$, then every α –open set is ii-open. Indeed, let M be α –open. If $int(M) = \phi$, then there is nothing to proof. So, $int(M) \neq \phi$ say $int(M) = K$, that is $M \in \tau^{int}$. Now, since M is α –open we have, $M \subseteq int(Cl(int(M)))$, and therefore, $M \subseteq Cl(int(M)) \subseteq Cl(M \cap K)$. Thus, $M \in \tau^i$. That is, $M \in \tau^{ii}$. Note that, the choosing $O \neq K$ in the definition of ii-open sets has no effect as shown in following example:

Let $X = \{0,1,2,3\}$ and $\tau = \{\phi, \{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3\}, X\}$.

$\tau^{int} = \{\phi, \{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3\}, \{0,2\}, \{0,3\}, \{0,2,3\}, X\}$.

$\tau^i = \{\phi, \{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3\}, \{1\}, \{2\}, \{3\}, \{0,2\}, \{0,3\}, \{1,2\}, \{1,3\}, \{0,2,3\}, \{1,2,3\}, X\}$.

$\tau^\alpha = \tau^{int} \neq \tau$, $\tau^{ii} = \tau^{int}$.

Conclusions

From above we concluded that ii-open set is a generalization of α –open set and it has the same property that the family of ii-open sets is always a topological space and there is no relation among the inter-open set and i-open set, while all ii-open sets are i-open and int-open.

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