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Evaluation of the Optical Characteristics of a Single Electrostatic Quadruple Lens Using the Transfer Matrix Method

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Abstract

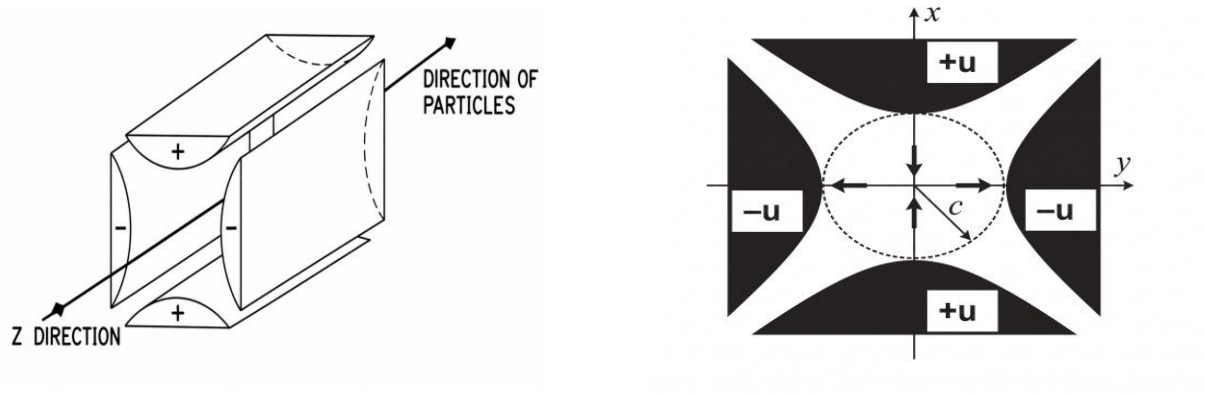
In the present work, a computational investigation carried out on the optical possession of an electrostatic quadruple lens. Calculations were performed using computer-based matrix methods, incorporating the "transport method." A rectangular-field model was employed to represent the potential distribution of the electrostatic quadruple lens. Furthermore, a program was developed in Fortran 90 to determine the various optical properties of this lens, based on tracing the trajectories of a beam (or stream of particles) along the lens axis. The analysis focused primarily on focal characteristics such as, focal planes, focal lengths, principal planes, magnification factors, and spherical aberration coefficients with in both the horizontal and vertical planes (corresponding to the converging and diverging planes, respectively). Additionally, the influence of specific parameters, such as the extraction coefficient and the effective length of the lens, on its overall performance was investigated.

Keywords: Electrostatic Quadruple, Optical Properties, Rectangular Model, Transfer Matrix Method.

1. Introduction

Quadruple lenses are commonly used for focusing charged particles. Photovoltaic devices marked the beginning of their use in the 1950s. Fixed quadruple lenses were used to reduce aberrations, as they act as corrective elements [1]. In recent years, quadruple lens systems have been incorporated into accelerators such as high-energy microprobes and scanning ion microscopes driven by the need to focus ions into an extremely small spot [2]. Electrostatic quadruple lenses are preferable to magnetic lenses when dealing with ion beams of moderate energy, this is because the focal length of electrostatic lenses, unlike that of their magnetic counterparts, does not depend on the mass of the charged particle. [3]. Devices produced in this way exhibit two levels of symmetry in addition to two levels of asymmetry, with the four poles arranged symmetrically on the two axes. Consequently, it has a focusing action for motion in one plan (either horizontal or vertical) and a defocusing action in the other plane. The particular characterization of their potential disposition is shown in Fig.1 (a-b). Where the pair of (A) The electrical potential of the electrodes is +V, while the other electrodes (B)

are at a potential of –V. The radius (C) defines the aperture of the channel formed by the four circular electrodes.



(a) (b)

Fig. (1): Electrostatic quadruple lens: (a) electrodes in the longitudinal direction, (b) electrodes in cross-section.

2. The Technique of Tracing Partial Trajectory using Matrices

1. The Trajectory Equation:

A beam of charged particles in Cartesian coordinates yields the trajectory equations for a quadruple lens field through which the particles pass [4].

$$x'' + \beta^2 f(z)x = 0 \quad (1)$$

$$y'' - \beta^2 f(z)y = 0 \quad (2)$$

Where (β) is the lens excitation and is given by

$$\beta^2 = V_1 K / C^2 V_0 \quad (3)$$

The electrode shape factor (K) is taken as, 1 [5]. The electrode potential is denoted by (V_1), and the accelerating potential by(V_0), while x" and y" represent the second derivatives with respect to the variable z. For the field distribution function of the quadruple lens, f(z), a rectangular field model of length (L) is used to approximate the actual performance of linear, straight quadruple lenses, as illustrated in Figure [6].

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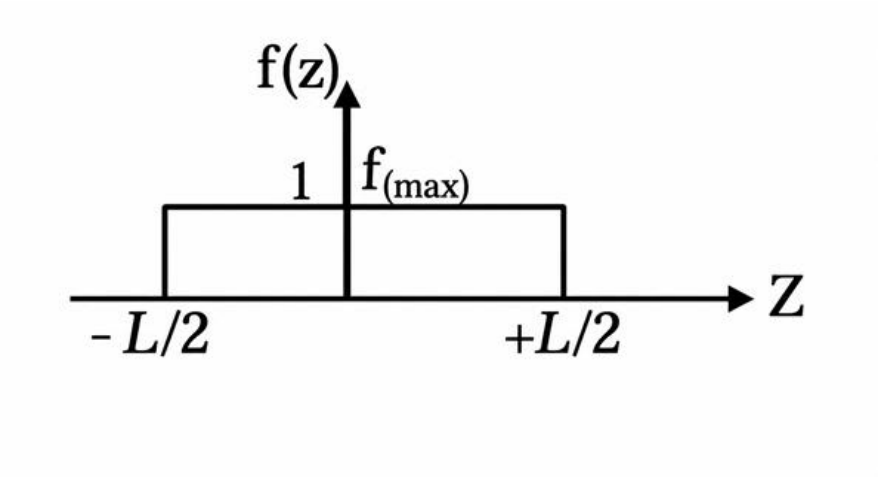


Fig. (2): The field distributions of a rectangular quadruple lens model [6]

This function can represented mathematically as follows [7]:

$$f(z) = f(z)_{max} = 1 \qquad \text{when} - L/2 \le Z \le L/2 \tag{4}$$

At point when $|z| > L/2$ the function $f(z) = 0$, The lens field, with an effective length equal to the length of the quadruple lens (L), is represented by the "square-top field division" model.

spherical aberration coefficients, defined by [7] :

$$\Delta x = M_C(C_{30}\, \propto^3 + C_{12}\alpha\gamma^2) \tag{5}$$

$$\Delta y = M_d(D_{03}\gamma^3 + D_{21}\gamma\alpha^2) \tag{6}$$

Spherical aberration and its coefficients are given by the equations below, where the coefficient (C) represents the aberration in the plane of convergence, while the coefficient (D) corresponds to the plane of divergence [8] .

Where $\mu = \beta u$, $\eta = 1 + \mu^2$, $\xi = 1 - \mu^2$.

$$C30/L = 1/16\{\eta 2 - (1 - 6\,\mu 2 + \mu 4)(\sin 4\theta/4\theta) + (u/L)\,\xi\,(1 - \cos 4\theta) + 1/3(2 - 2n + 3n2)[3\,\eta 2 - 4\,(1 - \mu 4)(\sin 2\theta/2\theta) + (1 - 6\,\mu 2 + \mu 4)(\sin 4\theta/4\theta) + 4(u/L)\,\eta\,(1 - \cos 2\theta) - (u/L)\,\xi\,(1 - \cos 4\theta)]\} \tag{7}$$

$$C12/L = 1/16\{6\,\eta\,\xi + 8\,(u/L) - [2\,\xi 2 + (5\,\xi\,\eta - 4\,\mu 2)\cos h\,2\theta](\sin 2\theta/2\theta) + [2\,\eta 2 - (\xi\,\eta + 20\,\mu 2)\cos 2\theta](\sin h\,2\theta/2\theta) - 2\,u/L\,(\xi\cos 2\theta - \eta\cosh 2\theta) + u/L\,(\eta - 5\,\xi)\sin 2\theta\sin h\,2\theta - u/L\,(5\,\eta + \xi)\cos 2\theta\cos h\,2\theta + (2 + 2n - n2)\{-2\,\eta\,\xi - 2\,u/L + [2\,\xi 2 - (\eta\,\xi - 4\,\mu 2)\cos h\,2\theta](\sin 2\theta/2\theta) + [2\,\eta 2 - (\eta\,\xi + 4\,\mu 2)\cos 2\theta](\sin h\,2\theta/2\theta) + 2\,u/L\,(\xi\cos 2\theta + \eta\cos h\,2\theta) + u/L(\eta - \xi)\sin 2\theta\sin h\,2\theta - u/L(\eta+\xi)\cos 2\theta\cosh 2\theta\}\} \tag{8}$$

The values , $\eta = 1 + \mu 2$, $\theta = \beta\,L$, $\xi = 1 - \mu 2$, $n = -1$,and $\mu = \beta\,u$ in quadruple lenses [9].

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2. The Transfer Matrix:

The transport matrices depict the change in the cross position and angle of a particle relative to the main beam axis. In the matrix form given below, which represents the solution to the two corresponding linear differential equations 1,2 [7]:

$$\begin{bmatrix} x(z) \\ x'(z) \end{bmatrix} = T_x \begin{bmatrix} x_0(z) \\ x'_0(z) \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} y(z) \\ y'(z) \end{bmatrix} = T_y \begin{bmatrix} y_0(z) \\ y'_0(z) \end{bmatrix} \quad (10)$$

The coefficients T_x and T_y represent, respectively, x'_0 and y'_0 the covariance matrices at the convergence and divergence levels (x-z) and, (y-z), along with the initial displacements x'_0 and y'_0 [10] [11] as in the equations below:

$$T_x = \begin{bmatrix} \cos(\beta L) & \frac{1}{\beta}\sin(\beta L) \\ -\beta\sin(\beta L) & \cos(\beta L) \end{bmatrix} \quad (11)$$

$$T_y = \begin{bmatrix} \cosh(\beta L) & \frac{1}{\beta}\sinh(\beta L) \\ \beta\sinh(\beta L) & \cosh(\beta L) \end{bmatrix} \quad (12)$$

In linear theory, a lens system is described using matrices. (m)

$$m = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad (13)$$

The elements a, b, c, and, d depend on the system, its geometry, and physical variables (focusing forces), rather than on the incident or emergent beam. By using a matrix, we can determine all the optical properties of the two planes in a single quadruple lens, such as:

1- The focal plane $Z = -\frac{a}{c}$ then,

$$Z_C = \frac{1}{\beta}\cot(\beta L) \quad \text{and} \quad Z_D = -\frac{1}{\beta}\coth(\beta L) \quad (14)$$

2- The focal distance $F = -\frac{1}{C}$

$$F_C = \frac{1}{\beta\sin(\beta L)} \quad \text{and} \quad F_D = -\frac{1}{\beta\sinh(\beta L)} \quad (15)$$

3- The principal plane $H = -(\frac{1-a}{c})$

$$H_C = \frac{-1-\cos(\beta L)}{\beta \sin(\beta L)} \quad \text{and} \quad H_D = \frac{1-\cosh(\beta L)}{\beta \sinh(\beta L)} \qquad (16)$$

4- The magnification $M = \frac{1}{cu+a}$

$$M_C = \frac{1}{\cos(\beta L) - u\beta L \sin(\beta L)} \text{ and } M_D = \frac{1}{u\beta L \sinh(\beta L) + \cosh(\beta L)} \qquad (17)$$

3. Result and Discussion

Positive ions move from left to right, specifically within a concentrated beam of charged particles is taken into consideration by single electrostatic quadruple lens. For a rectangular field with an effective length equal to the quadruple's length (L), The track equation of charged particles has been resolved [12]. The object is assumed to be located at a position (u) along the axis at the beginning of the lens region, while the image is formed at a distance (v) from the end of that region. The trajectory of a beam of charged particles accelerated along the optical axis (z) is influenced by the geometric parameters (L), (u), and (v), as well as the lens excitation (β).

The trajectory of a charged-particle beam through an electrostatic quadruple lens in both converges and diverging lens planes respectively as shown in figure (3). It should be made clear that, in the convergence plane (x-z plane), The repulsive force resulting from a positive potential (+V) acts upon positively charged particles when applied to the lens electrodes located along the x-axis, effectively directing them toward the optical axis (z-axis); once they cross this axis, the quadruple lens causes the beam to diverge in the conjugate planes [13].

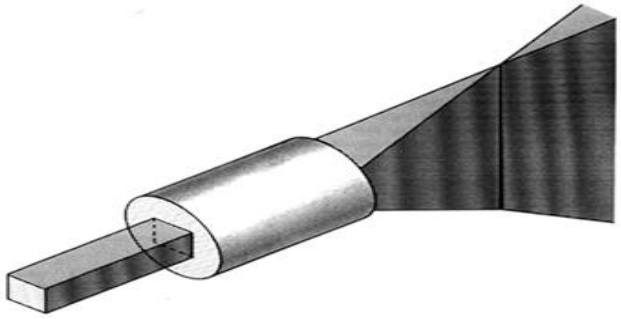


Fig. (3): A single quadruple lens showing its effect on a beam of charged particles; horizontal convergence and vertical divergence form a linear image [14].

As for the (y-z plane) divergence, along this axis, positively charged particles are attracted toward the negative potential (-V), which directs them away from the (z) axis; thus, a quadruple lens cannot act as a focusing device it functions instead as a diverging one. Consequently, the focusing process is concentrated along the (x) axis, while the diverging process is concentrated along the (y) axis [15].

The paths are drawn according to the following initial conditions: $x_0 = 1, x_0' = 0, y_0 = 1, y_0' = 0$. The results are in agreement with those reported by Baranova and Yavor in [13]. A computer program was developed in Fortran 90, based on the transfer matrix method and utilizing equations (8–11), to calculate the focal properties of a quadrupole lens. The program investigates how variations in the excitation parameter (β) and the lens length (L) affect the beam trajectory and, consequently, the lens's focal characteristics. Table (1) presents the calculated focal parameters for the quadruple lens at various excitation levels, with a fixed length of L = 5 mm and distances u and v both set to 20 mm.

Table (1): Calculating the effect of various excitation parameters on the calculated focal parameters of a quadruple lens. Where L = 5 mm and u=v=20 mm.

β $(mm)^{-1}$	ZC	FC	HC	MC	QC	ZD	FD	HD	MD	QD	XX	XY
0.111	1621.58	1624.08	-2.50	1.01	-25.32	-1624.91	-1622.41	-2.50	0.99	-24.69	0.068	0.062
0.222	404.14	406.65	-2.50	1.06	-26.32	-407.48	-404.98	-2.50	0.95	-24.69	0.068	0.06
0.333	178.69	181.20	- 2.51	1.14	-28.20	-182.02	-179.53	-2.49	0.89	-22.48	0.2	0.087
0.444	99.78	102.29	-2.51	1.28	-31.37	103.11	-100.62	-2.49	0.82	-20.87	0.47	0.14
0.555	63.25	65.77	-2.52	1.52	-36.75	-66.59	-64.27	-2.48	0.74	-19.13	1.14	0.25
0.666	43.41	45.93	-2.52	1.96	-46.72	-46.74	-44.27	-2.48	0.66	-17.38	2.77	0.43
0.777	31.44	33.98	-2.53	2.97	-69.43	-69.43	-32.31	-2.47	0.59	-15.72	7.3	0.68

From this table it clear that (F_C) has always positive sign while, (F_D) has always negative sign this mean that the quadruple lens is always they converge in the x-z plane and diverge in the y-z plane.

Figure (4) it illustrates the relationship between the focal disperser (FD) and the focal dimensions (FC), expressed in terms of the excitation parameter (β). It has been found that both (FC) and (FD) decrease as β increases.

Parametric analysis revealed that the lens excitation parameter (β) or the effective electrode length (L) significantly increases the lens's refractive power. As shown in the figure, this results in a decrease in both the diverging and converging focal lengths (FD and FC), accompanied by a consistent shift of the focal and principal planes toward the center of the lens [16].

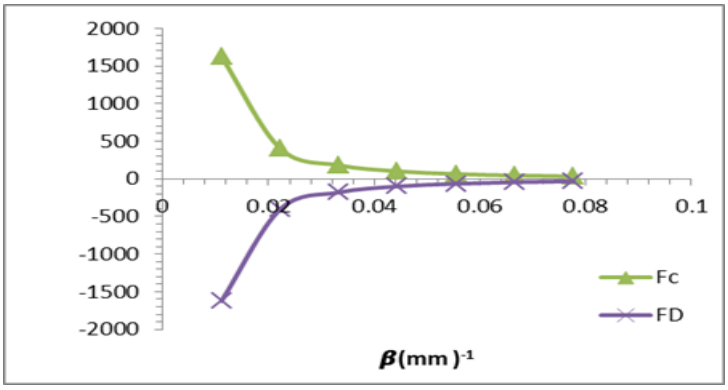


Fig. (4): The focal lengths FC and FD in the convergence and divergence plane as a β in an electrostatic quadruple lenses of length 5mm.

Figure (5) shows the relation between the magnification (MC) and (MD) in both converging and diverging planes respectively as a (β), it shows that (MC) increasing with increasing (β) while, (MD) decreasing when (β) increasing.

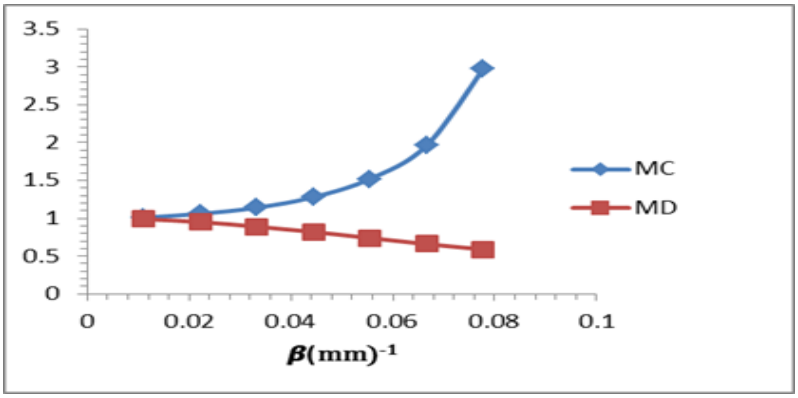


Fig. (5): The relation between the magnification (MC) and (MD) in both converging and diverging planes respectively as a (β).

While the divergence plane magnification (MD) decreases monotonically with increasing excitation and electrode length due to the hyperbolic nature of the defocusing transfer matrix, the convergence-plane magnification (MC) exhibits a pronounced trigonometric dependence on the dimensionless focusing strength (βL), rising sharply before undergoing a sign reversal at L mm (as the beam transitions from a virtual to a real inverted imaging regime. Moreover, the rapid escalation of the spherical aberration figures at elevated excitation levels and near the asymptotic magnification threshold underscores a critical design trade-off between achieving short focal lengths and maintaining minimal geometric beam distortion, thereby providing essential quantitative guidelines for optimizing electrostatic quadruple lenses in ion microprobe and beam-transport applications [17].

The spherical aberration XA and YA in a conversing plan x-z, y-z planes, consecutively, as a function of β for an electrostatic quadruple lens of length 5mm is shown in figure (6).

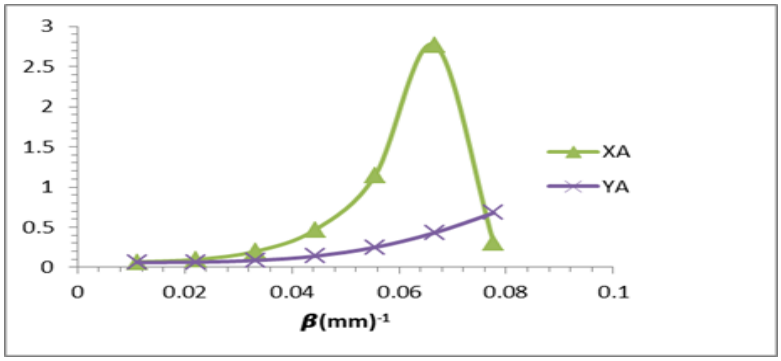


Fig. (6): Spherical aberration in a conversing plane in the x-z and y-z planes, as a (β).

The effect of different the length of the quadruple lens on the focal properties of the quadruple lens was studied under a (β=0.0555mm-1) and by keeping u=v=20mm.show from table (2).

Table (2): The calculated focal parameters of the quadruple lens under various lengths of the lens (L).

L (mm)	ZC	FC	HC	MC	QC	ZD	FD	HD	MD	QD	XA	YA
5	63.25	65.77	-2.52	1.52	-36.75	-66.59	- 64.10	-2.48	.74	-19.13	1.14	0.25
10	29.06	34.19	-5.13	3.77	-99.97	-35.73	-30.86	-4.88	.55	-18.65	3.86	0.31
15	16.40	24.36	-7.97	-6.76	181.06	-26.43	-19.33	-7.09	.42	-18.38	-9.12	0.43
20	8.94	20.12	-11.17	-1.82	45.55	-22.41	-13.32	-9.09	.31	-18.22	-3.1	0.64
25	3.34	18.32	-14.98	-1.10	23.50	-20.41	-9.60	-10.82	.24	-18.14	-2.27	1.07
30	-1.70	18.10	-19.80	-0.83	13.39	-19.36	-7.07	-12.28	.18	-18.09	-1.94	2.00

The relation between the focal length FC and FD in both converging and diverging planes, respectively, as a function of length of lens (L) is shown in figure (7). It should be clear from this figure that the FC is always positive and decrease as the length (L) is increases while, FD is always negative and decreases as the (L) is increases.

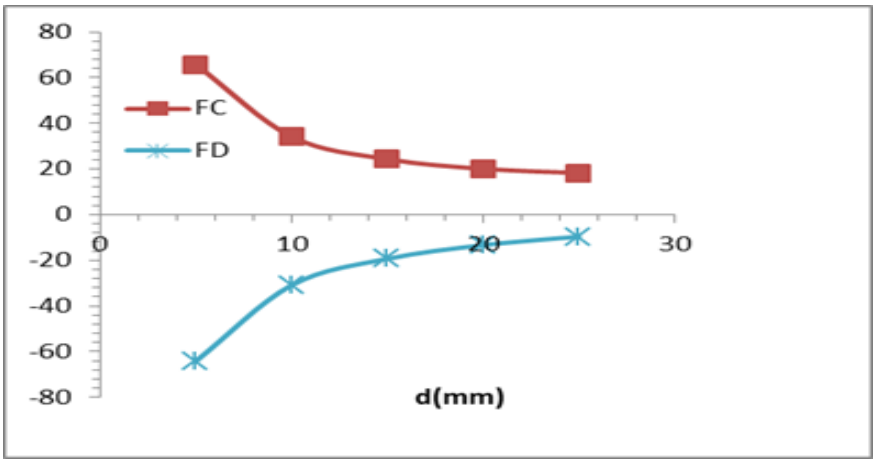


Fig. (7): The relation between the focal length FC and FD in both converging and diverging planes, respectively, as a (L).

Relation between the magnification MC and MD in converging and diverging planes, respectively, as a (L) is shown in figure (8). It is clear from this figure that the MC increases as the length (L) increases till 13 mm, then decreases with a negative sign as (L) increases. While MD always decreases as the (L) is increases. This behavior corresponds to the behavior of the element of the transfer matrix. Its clear from this figure 8 the value of XA increases from 1.1 at d = 5 mm to 3.9 at d = 10 mm. Subsequently, it becomes negative, reaching about -9.1 at d = 15 mm, before its absolute value decreases to about 2.3 at d = 25 mm. In contrast, the value of YA remains positive, increasing gradually from about 0.2 to 1.0 across the studied range.

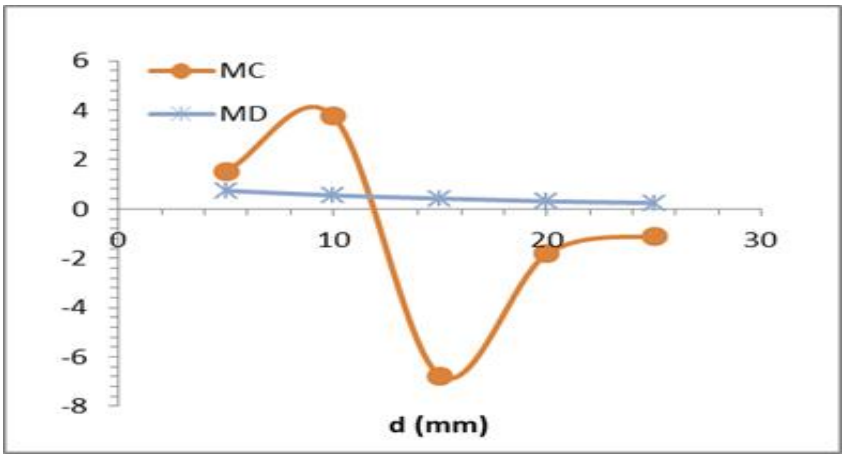


Fig. (8): The relation between the magnification MC and MD in converging and diverging planes, respectively, as a (L).

The spherical aberration XA and YA in a converging plan x-z, y-z planes, consecutively, as a function of lens length L is shown in Figure 9. This figure shows that the spherical aberration coefficients, as a function of lens length, decrease, approaching values that are near-ideal [18].

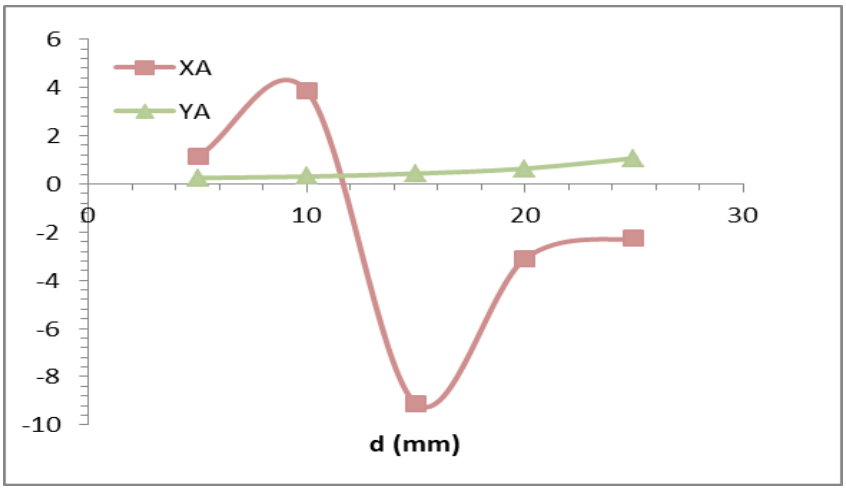


Fig. (9): Spherical aberration in a conversion plan x-z, y-z plans, as a function of the length of the lens (L).

4. Conclusion

This study demonstrates numerical efficiency and reliability in evaluating first-order fundamental elements and the properties of a single electrostatic quadruple lens. Using the Fortran 90 programming language, it is shown that a beam of charged particles simultaneously undergoes concurrence Separation in the vertical plane and convergence in the horizontal plane.

The first-order optical properties of a single electrostatic quadruple lens are calculated using the

transfer matrix method and a rectangular field model. The results explain the astigmatic effect inherent in such a lens. For a given electrode polarity, positive ions are the x–z plane is focused, while the y–z plane is unfocused, Furthermore, an increase in the excitation parameter or the effective length of the lens leads to a reduction in the focal length within the focusing plane, thereby enhancing the lens's effect in both planes.

The dependence of the image -plane magnification on the lens length underscores the critical importance of carefully selecting operating parameters. Given values of $\beta = 0.0555\ mm^{-1}$ and u = 20 mm, the ideal model predicts a singularity in the magnification at the focal plane when the lens length L =13.21 mm, with the sign of the magnification changing as this point is crossed. Furthermore, the results highlight the combined significance of excitation strength and effective length in defining desired focusing characteristics, this provides a foundation for the design of the quadruple lenses upon which beam-transport systems rely. Future studies should incorporate an analysis of non-linear trajectories alongside realistic field distributions, which is essential for obtaining a more comprehensive assessment of practical optical performance.

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