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Estimations on Rician Distribution Using Ranked Set Sampling

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Abstract

There are many situations where it is costly to measure the variable under the study, but this variable can be easily ranked. Ranked set sampling (RSS) utilizes the information of the ranks in addition to the measurements. It was proved that RSS is more efficient than simple random sampling (SRS) when estimating the population arithmetic mean for the same sample size. Because of its efficiency, RSS was employed to a wide range of applications and becoming an interesting method for many researchers. It was studied parametrically and non-parametrically and several estimators based on different statistical distributions were investigated. This research adopts Rician distribution and several measurements from this distribution were investigated based on RSS. The estimations include the arithmetic, geometric, harmonic, quadratic means as well as the median, variance, mean deviation, coefficient of variation, coefficient of skewness and kurtosis. Simulation study shows that the estimators based on RSS are more efficient than those based on SRS. However, the gain in the efficiency with variance and coefficient of kurtosis is not too much and even worse for small sample size.

Keywords: Ranked Set Sampling, Simulation, Rician Distribution, Arithmetic Mean, Geometric Mean, Harmonic Mean, Quadratic Mean, Mean Deviation, Coefficient of Variation, Coefficient of Skewness, Coefficient of Kurtosis, Biasness,

MSE and Relative Efficiency.

1. Introduction

The procedure of RSS can be carried out as follows:

1. Select Randomly m^2 simple random sample (SRS) units from the population.
2. Allocate the m^2 selected units as randomly as possible into m sets each of size m.
3. Rank the units within each set before measurement by any possible mean like personal judgment or a concomitant variable or any cheap instrument.
4. Choose for actual measurement the smallest ranked unit in the first set, then the second smallest ranked unit in the second set, continuing in this way till the largest ranked unit is selected from the last set.
5. Repeat the above steps r times till obtain the required sample of size $n = rm$.

The Rician distribution, also known as the Rice distribution, is a probability distribution used in statistics to model random variables that have both magnitude and phase components. It was named after the British radio engineer Harold Rice. The probability density function (pdf) for Rician distribution is given by

$$f(x|v,\sigma) = \frac{x}{\sigma^2} exp\left(\frac{-(x^2+v^2)}{2\sigma^2}\right) I_0\left(\frac{xv}{\sigma^2}\right), \qquad 0 \le x < \infty$$

$v\ and\ \sigma^2$ are two parameters and $I_\circ(\frac{xv}{\sigma^2})$ is the modified Bessel function of the first kind with order zero. See (Rice (1944), Proakis (1995), Rappaport (2002), Poon, et.al (2004), Behrens & Joshi (2013), Goldsmith (2005) and Yerima & Sparkes (2015)). Figure 1 shows the distribution for different values of the parameter v and $\sigma = 1$.

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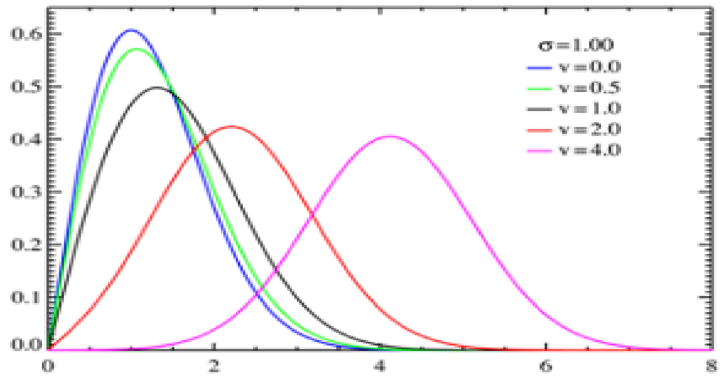

Figure (1): probability density function for Rician distribution with different parameters.

In our study, we focus on ranked set sampling (RSS) and the samples are drawn from Rician distribution. So, we investigate several statistics: arithmetic mean, geometric mean, harmonic mean, quadratic mean, median, mean deviation, variance, coefficient of skewness and kurtosis. The relative efficiency of these statistics obtained by RSS are compared with those obtained by simple random sampling (SRS). Computer simulation is used to investigate the properties of these statistics.

2. Literature Review

RSS technique is useful for cases when the variable of interest can be more easily ranked than quantified. The method was proposed by McIntyre (1952) to improve the method of estimation of a population mean. The terminology of RSS was coined by Halls and Dell (1966). RSS has been applied by many researchers and showed that RSS estimators are more efficient than SRS. Halls and Dell (1966) evaluated the performance of RSS for estimating the weights of browse and herbage in a pine-hardwood forest of east Texas USA. They found that RSS was more efficient than SRS. Evans (1967) conducted a study on an area seeded to longleaf pine in central Louisiana and showed that RSS gave smaller MSE than SRS. Takahasi and Wakimoto (1968) considered the length of a kind of bacterial cell in a microscopic field, measured using a micrometer. Several researches were done based on specific distributions. RSS was shown to perform better to estimate the mean and the parameters as well as distributions. The following are some of the useful studies; Fei et al. (1994) studied RSS to estimate the parameters of Weibull distribution. Lam et al. (1994) estimated the location, scale and quantile of the exponential distribution using RSS. Shen (1994) estimated the mean of a lognormal distribution with a known coefficient of variation using RSS. Stokes (1995) estimated the maximum likelihood estimator and the best linear unbiased estimation for the location-scale family of random variables using RSS. Kaur et al. (1997) considered the unequal allocations using RSS for skew distributions. Adatia (2000) generalized RSS and used it to estimate the location and scale parameters of the half-logistic distribution. Al-Saleh et al. (2000) investigate Bayesian estimation of the parameter of the underlying distribution using RSS. Kaur and Taillie (2000) studied optimal RSS allocations for two classes of symmetric distributions. Al-Hadhrami (2009) studied ratio type estimators of the population mean based on RSS. Al-Omari, et.al (2009) considered multistage balanced groups RSS for estimating the population median. Al-Hadhrami (2010) investigated estimation of the population variance using RSS with auxiliary variable. Al-Hadhrami (2010) studied parametric estimation on modified Weibull distribution based on RSS. Al-Hadhrami (2010) studied chain ratio type estimators based on several methods. Recent research on RSS includes the followings: Melek & Selma (2017) studied parameter estimation of generalized Rayleigh distribution based on ranked set sample based on RSS. Samuh, et al. (2020) considered estimation of the parameters of the new Weibull-Pareto distribution using RSS. Bantan, et al. (2020) investigated Properties and estimation of Lomax distribution based on RSS.

Although there are many studies on RSS, the research on Rician distribution using RSS remains limited. In addition, the research on geometric, harmonic and quadratic means, coefficient of skewness and kurtosis estimators using RSS are very rare.

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3.Study Design and Research Method

The study is based on Monte Carlo simulation using R package programming and was carried out with 100000 number of iterations. The simulation is used in order to investigate the properties of RSS estimators. The following is the algorithm to generate a random sample from Rician distribution:

1. Generate $X \sim N(\upsilon.cos\theta, \sigma^2)$.
2. Generate $Y \sim N(\upsilon.sin\theta, \sigma^2)$. X and Y are independent.
3. $R = \sqrt{X^2 + Y^2}$. $R \sim Rice(|\upsilon|, \sigma)$

Another possible algorithm is as follows:

1. Generate $P \sim Poisson\left(\frac{\upsilon^2}{2\sigma^2}\right)$.
2. Generate $X \sim \chi^2_{(2P+2)}$.
3. $R = \sigma\sqrt{X}$.

The algorithm for the whole program for all estimators:

1. Generate a random sample from Rician distribution using both simple random sampling (SRS) and ranked set sampling (RSS). The sample size $n = mr$ where m is the set size and r is number of cycles.
2. The estimators are calculated for each sample.
3. Repeat steps (1) and (2) k times. The value of k represents the number of iterations.
4. Calculate the average of the values of the estimates as an estimate of the expectation of both SRS and RSS estimators. Thus, obtain the bias as the difference between the parameter and the expected value of the corresponding estimator.
5. Calculate the variances of the estimators for both SRS and RSS. Then, the MSE is obtained by summing the bias squared and the variance of the estimator.
6. Calculate the relative precision of RSS estimators by finding the ratio of the MSE

of the estimator based on SRS and the MSE of the corresponding estimator based on RSS.

7. The process is repeated for different set size and number of cycles.

4. Results and Discussion

4.1 Results About the Arithmetic Mean:

The arithmetic mean is the most common measurement of central tendency when the data are quantitative. It summarizes huge values of data by a single value that is the ratio of the sum of all observations to the total number of observations.

The arithmetic mean possesses some nice properties that make it desirable by researchers and is used in diverse fields. It is simple to understand and easy to calculate and retains the measurement unit of the original data. It utilizes the information in every single observation in the data. It is embedded in many other statistical measurements such as the variance, mean deviation, coefficients of skewness and kurtosis, Pearson correlation coefficients. It is widely used in inferential statistics to compare groups of quantitative data.

The arithmetic mean of Rician distribution is given by:

$$\mu = \sigma\sqrt{\frac{\pi}{2}}L_{1/2}\left(\frac{-v^2}{2\sigma^2}\right)$$

Where $L_{1/2}\left(\frac{-v^2}{2\sigma^2}\right)$ is the square of Laguerre polynomial. The general formula of this polynomial is given by:

$$L_n(x) = \sum_{r=0}^{n}(-1)^r \frac{n!}{(n-r)!\,(r!)^2}x^r.$$

The simulation results are shown in Table 1 for skewed case and Table 2 for symmetric case. The tables show the amount of bias, the variances of the two estimators and MSEs. The relative precision is given in the last column.

Table (1): Simulation outcomes to estimate the arithmetic mean using RSS and SRS from Rician distribution with parameters $\sigma = 4, v = 10$.

Set size	cycle	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.0017	0.0114	2.5044	4.7808	2.5044	4.7810	1.91
4	1	0.0038	0.0129	1.5159	3.5935	1.5159	3.5937	2.37
5	1	0.0005	0.0004	1.0342	2.8665	1.0342	2.8665	2.77
6	1	0.0038	0.0063	0.7457	2.4199	0.7457	2.4200	3.25
3	2	0.0054	0.0002	1.2502	2.4075	1.2503	2.4075	1.93
4	2	0.0001	0.0003	0.7520	1.8002	0.7520	1.8002	2.39
5	2	0.0027	0.0001	0.5130	1.4316	0.5130	1.4316	2.79
6	2	0.0000	0.0074	0.3716	1.2029	0.3716	1.2030	3.24
3	3	0.0032	0.0009	0.8322	1.5994	0.8322	1.5994	1.92
4	3	0.0050	0.0023	0.5103	1.2095	0.5103	1.2095	2.37
5	3	0.0038	0.0029	0.3437	0.9582	0.3437	0.9582	2.79

From the simulation results summarized in Table 1 and Table 2, we notice the following:

- The estimators are unbiased and the fluctuation is due to simulation error.
- The variances of the estimators decrease as the sample size increases.
- The variance and the MSE of RSS estimator are less than the counterpart using SRS and thus RSS estimators are more efficient than SRS estimator.
- The relative efficiency shown on the last column increases as the sample size increases.

Table (2): Simulation outcomes to estimate the arithmetic mean using RSS and SRS from Rician distribution with parameters $\sigma = 1,\ v = 14.5$ (symmetric case)

Set size	cycle	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.0006	0.0057	0.1741	0.3307	0.1741	0.3307	1.90
4	1	0.0004	0.0041	0.1051	0.2495	0.1051	0.2495	2.37
5	1	0.0025	0.0002	0.0718	0.2015	0.0718	0.2015	2.81
6	1	0.0009	0.0020	0.0521	0.1655	0.0521	0.1655	3.18
3	2	0.0008	0.0017	0.0867	0.1671	0.0867	0.1671	1.93
4	2	0.0018	0.0005	0.0532	0.1250	0.0532	0.1250	2.35
5	2	0.0012	0.0023	0.0360	0.1004	0.0360	0.1004	2.79
6	2	0.0005	0.0004	0.0261	0.0825	0.0261	0.0825	3.15
3	3	0.0027	0.0009	0.0571	0.1110	0.0571	0.1110	1.95
4	3	0.0007	0.0002	0.0355	0.0825	0.0355	0.0825	2.33
5	3	0.0005	0.0017	0.0239	0.0660	0.0239	0.0660	2.76
6	3	0.0003	0.0021	0.0175	0.0555	0.0175	0.0555	3.17

4.2 Results about Geometric Mean:

Geometric mean is a measure of central tendency and defined by the n^{th} root of the product of the observations. $G = \sqrt[n]{x_1 . x_2 \dots x_n}$

It is only suitable for positive observations to avoid negative value under the root. It may be used to obtain the average of ratio and percentages such as the growth of a population, financial indices and interest rate.

The simulation outcomes are summarized in Table 3 for skewed case and Table 4 for symmetric case.

Table (3): Simulation outcomes to estimate the geometric mean using RSS and SRS from Rician distribution with parameters $\sigma = 4, v = 10$.

set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.1641	0.2755	3.1806	5.4449	3.2075	5.5208	1.72
4	1	0.1099	0.2026	2.0447	4.1970	2.0567	4.2380	2.06
5	1	0.0741	0.1757	1.4512	3.3860	1.4567	3.4169	2.35
6	1	0.0594	0.1516	1.0824	2.8876	1.0859	2.9106	2.68
3	2	0.0911	0.1489	1.6610	2.8758	1.6693	2.8979	1.74
4	2	0.0543	0.1119	1.0534	2.1780	1.0563	2.1905	2.07
5	2	0.0422	0.0886	0.7440	1.7499	0.7458	1.7578	2.36
6	2	0.0281	0.0641	0.5557	1.4735	0.5565	1.4776	2.66
3	3	0.0549	0.1003	1.1225	1.9455	1.1255	1.9555	1.74
4	3	0.0293	0.0750	0.7258	1.4839	0.7267	1.4896	2.05
5	3	0.0318	0.0582	0.5017	1.1821	0.5027	1.1855	2.36
6	3	0.0187	0.0537	0.3748	0.9837	0.3752	0.9866	2.63

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Table (4): Simulation outcomes to estimate the geometric mean using RSS and SRS from Rician distribution with parameters $\sigma = 1, \upsilon = 14.5$ (symmetric case)

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.0054	0.0058	0.1755	0.3323	0.1755	0.3324	1.89
4	1	0.0041	0.0128	0.1063	0.2509	0.1063	0.2510	2.36
5	1	0.0002	0.0067	0.0727	0.2026	0.0727	0.2026	2.79
6	1	0.0028	0.0078	0.0527	0.1665	0.0527	0.1666	3.16
3	2	0.0038	0.0041	0.0875	0.1682	0.0875	0.1682	1.92
4	2	0.0037	0.0048	0.0538	0.1258	0.0538	0.1259	2.34
5	2	0.0025	0.0058	0.0364	0.1010	0.0364	0.1011	2.78
6	2	0.0004	0.0033	0.0265	0.0830	0.0265	0.0830	3.13
3	3	0.0007	0.0047	0.0577	0.1116	0.0577	0.1117	1.94
4	3	0.0005	0.0030	0.0359	0.0831	0.0359	0.0831	2.32
5	3	0.0004	0.0006	0.0242	0.0664	0.0242	0.0664	2.74
6	3	0.0003	0.0001	0.0178	0.0559	0.0178	0.0559	3.15

From the Table 3 and Table 4 we may conclude the followings:

- In case of asymmetry, both geometric means from SRS and RSS are biased but the bias decreases as sample size increases. The amount of bias in RSS estimator is less than for SRS. In addition, the amount of bias when the distribution is symmetric is smaller than with skewed case.
- The variances of the estimators decrease as the sample size increases. The variance and the MSE of RSS estimator are always less than the corresponding one using SRS. and thus, RSS estimator is more efficient than SRS estimator.
- The relative efficiency of RSS estimator with respect to SRS increases as the set size increases. The relative efficiency when the distribution is symmetric is higher than the case of skewed distribution.

4.3 Results about Harmonic Mean:

The harmonic mean is used to obtain the average involving ratio and rates. It is defined as the reciprocal of the arithmetic mean of the reciprocals of the observations. It can be expressed in the following formula

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$$H = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}}$$

The observations must be non-zero to avoid division by zero. Computer simulations are summarized in Table 5 for skewed case and Table 6 for symmetric case.

Table (5): Simulation outcomes to estimate the harmonic mean using RSS and SRS from Rician distribution with parameters $\sigma = 4, v = 10$.

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.7168	0.9477	4.4376	6.5189	4.9514	7.4171	1.50
4	1	0.5643	0.7750	3.2992	5.3676	3.6176	5.9682	1.65
5	1	0.4607	0.6823	2.6623	4.5649	2.8745	5.0305	1.75
6	1	0.3995	0.6050	2.2290	4.0688	2.3885	4.4348	1.86
3	2	0.4778	0.6064	2.8719	4.0490	3.1002	4.4167	1.42
4	2	0.3660	0.4967	2.1733	3.3123	2.3072	3.5590	1.54
5	2	0.3091	0.4213	1.7706	2.8370	1.8661	3.0144	1.62
6	2	0.2586	0.3564	1.5074	2.5216	1.5742	2.6485	1.68
3	3	0.3626	0.4577	2.2237	3.0527	2.3552	3.2622	1.39
4	3	0.2714	0.3707	1.7421	2.5203	1.8157	2.6577	1.46
5	3	0.2448	0.3180	1.3872	2.1400	1.4471	2.2411	1.55
6	3	0.2023	0.2811	1.1877	1.8839	1.2286	1.9629	1.60

Table (6): Simulation outcomes to estimate the harmonic mean using RSS and SRS from Rician distribution with parameters $\sigma = 1, v = 14.5$ (symmetric case)

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.0116	0.0176	0.1783	0.3351	0.1784	0.3354	1.88
4	1	0.0080	0.0218	0.1085	0.2532	0.1085	0.2536	2.34
5	1	0.0029	0.0138	0.0745	0.2044	0.0745	0.2046	2.75
6	1	0.0048	0.0137	0.0540	0.1681	0.0540	0.1683	3.12
3	2	0.0070	0.0100	0.0892	0.1699	0.0892	0.1700	1.90
4	2	0.0058	0.0092	0.0549	0.1272	0.0550	0.1273	2.32
5	2	0.0038	0.0094	0.0373	0.1021	0.0373	0.1022	2.74
6	2	0.0012	0.0063	0.0272	0.0839	0.0272	0.0840	3.09
3	3	0.0015	0.0086	0.0589	0.1128	0.0589	0.1128	1.92
4	3	0.0018	0.0060	0.0367	0.0841	0.0367	0.0841	2.29
5	3	0.0013	0.0030	0.0248	0.0671	0.0248	0.0671	2.70
6	3	0.0009	0.0019	0.0183	0.0566	0.0183	0.0566	3.10

From Table 5 and Table 6, the following points are worth to notice:

- Both harmonic means from SRS and RSS are biased but the bias decreases as the sample size increases. In addition, RSS estimator has less bias then SRS estimator. Moreover, the amount of bias when the distribution is symmetric is less than that when the distribution is asymmetric.
- For both estimators, their variances and MSEs decrease as the sample size increases. The values of the variance and MSE of RSS estimator is smaller than that from SRS. Thus, RSS estimator performs better than the corresponding one using SRS.
- The relative efficiency of RSS estimator with respect to SRS estimator is greater than 1 and increases as the set size increases. In addition, the relative efficiency when the distribution is symmetric is higher than the case of skewed distribution.

4.4 Results about Quadratic Mean:

Quadratic mean or root mean square is a type of mean defined as the square root of the average of the squares of observations. It can be expressed as

$$Q = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}$$

Simulation results are provided in Table 7 for skewed case and in Table 8 for symmetric case.

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Table (7): Simulation outcomes to estimate the quadratic mean using RSS and SRS from Rician distribution with parameters $\sigma = 4, v = 10$.

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.1084	0.2193	2.5606	4.7150	2.5724	4.7631	1.85
4	1	0.0654	0.1665	1.5703	3.5282	1.5746	3.5559	2.26
5	1	0.0482	0.1251	1.0891	2.8187	1.0914	2.8344	2.60
6	1	0.0299	0.0956	0.7968	2.3740	0.7977	2.3832	2.99
3	2	0.0508	0.1038	1.2835	2.3595	1.2860	2.3703	1.84
4	2	0.0350	0.0780	0.7839	1.7632	0.7852	1.7693	2.25
5	2	0.0217	0.0609	0.5417	1.4012	0.5422	1.4049	2.59
6	2	0.0172	0.0573	0.3977	1.1821	0.3980	1.1854	2.98
3	3	0.0409	0.0678	0.8556	1.5651	0.8573	1.5697	1.83
4	3	0.0275	0.0482	0.5310	1.1852	0.5318	1.1876	2.23
5	3	0.0133	0.0448	0.3644	0.9379	0.3646	0.9399	2.58
6	3	0.0123	0.0311	0.2660	0.7805	0.2662	0.7815	2.94

Table (8): Simulation outcomes to estimate the quadratic mean using RSS and SRS from Rician distribution with parameters $\sigma = 1, v = 14.5$ (symmetric case)

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.0065	0.0171	0.1741	0.3301	0.1741	0.3304	1.90
4	1	0.0032	0.0045	0.1051	0.2489	0.1051	0.2490	2.37
5	1	0.0051	0.0070	0.0717	0.2011	0.0717	0.2012	2.81
6	1	0.0009	0.0038	0.0522	0.1652	0.0522	0.1653	3.17
3	2	0.0022	0.0074	0.0866	0.1668	0.0866	0.1668	1.93
4	2	0.0002	0.0038	0.0532	0.1248	0.0532	0.1248	2.35
5	2	0.0000	0.0012	0.0360	0.1002	0.0360	0.1002	2.79
6	2	0.0013	0.0025	0.0262	0.0823	0.0262	0.0823	3.15
3	3	0.0047	0.0028	0.0570	0.1109	0.0570	0.1109	1.94
4	3	0.0019	0.0026	0.0355	0.0824	0.0355	0.0824	2.32
5	3	0.0013	0.0039	0.0240	0.0660	0.0240	0.0660	2.75
6	3	0.0008	0.0039	0.0175	0.0553	0.0175	0.0554	3.16

The following remarks are extracted from Table 7 and Table 8:

- The estimators of the quadratic mean are biased and the amount of bias becomes smaller as the sample size gets bigger. The estimators are asymptotically consistent. Also, the amount of bias when the distribution is symmetric is smaller than the case when the distribution is skewed.
- The variance and MSE decrease when sample size increases. They are also

smaller for RSS estimator than from SRS.

- The relative efficiency of RSS estimator with respect to SRS estimator is bigger than 1 and increases as the set size increases. Hence, the quadratic mean estimator based on RSS estimator performs better than the that from SRS.
- The relative efficiency of the RSS estimator is higher when the distribution is symmetric.

4.5 Results about the Median:

Another measure of central tendency is the median. It is a single value occurred in the middle of ranked data. That is half of the observations below the median and half of them are below it. It can also be seen as the 50th percentile of the data. Simulation outcomes are given in Table 9 for skewed case and Table 10 for symmetric case.

Table (9): Simulation outcomes to estimate the median using RSS and SRS from Rician distribution with parameters $\sigma = 4, v = 10$.

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.0168	0.0066	4.0874	6.5703	4.0877	6.5704	1.61
4	1	0.0182	0.0059	2.0059	4.3954	2.0063	4.3954	2.19
5	1	0.0094	0.0140	2.0097	4.2136	2.0098	4.2138	2.10
6	1	0.0115	0.0178	1.1711	3.1930	1.1712	3.1933	2.73
3	2	0.0128	0.0087	1.8261	3.1760	1.8263	3.1761	1.74
4	2	0.0096	0.0015	1.2218	2.4845	1.2219	2.4845	2.03
5	2	0.0055	0.0049	0.9038	2.0363	0.9038	2.0363	2.25
6	2	0.0023	0.0006	0.6999	1.7410	0.6999	1.7410	2.49
3	3	0.0011	0.0054	1.5369	2.4526	1.5369	2.4527	1.60
4	3	0.0009	0.0107	0.8984	1.7461	0.8984	1.7463	1.94
5	3	0.0040	0.0051	0.7328	1.5004	0.7328	1.5004	2.05
6	3	0.0001	0.0097	0.5023	1.1939	0.5023	1.1940	2.38

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Table (10): Simulation outcomes to estimate the median using RSS and SRS from Rician distribution with parameters $\sigma = 1, v = 14.5$ (symmetric case)

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.0020	0.0029	0.2776	0.4448	0.2776	0.4449	1.60
4	1	0.0000	0.0019	0.1353	0.2973	0.1353	0.2973	2.20
5	1	0.0027	0.0017	0.1371	0.2881	0.1371	0.2881	2.10
6	1	0.0002	0.0023	0.0784	0.2134	0.0784	0.2134	2.72
3	2	0.0007	0.0002	0.1231	0.2159	0.1231	0.2159	1.75
4	2	0.0010	0.0004	0.0827	0.1688	0.0827	0.1688	2.04
5	2	0.0015	0.0024	0.0612	0.1389	0.0612	0.1389	2.27
6	2	0.0017	0.0004	0.0477	0.1163	0.0477	0.1163	2.44
3	3	0.0012	0.0000	0.1024	0.1666	0.1024	0.1666	1.63
4	3	0.0010	0.0012	0.0599	0.1161	0.0599	0.1161	1.94
5	3	0.0009	0.0023	0.0492	0.1015	0.0492	0.1015	2.06
6	3	0.0006	0.0020	0.0341	0.0809	0.0341	0.0809	2.37

Based on simulation results, the following outcomes are observed:

- The bias of RSS estimators is very little and the amount of bias becomes smaller as the sample size gets bigger. Also, the amount of bias when the distribution is symmetric is less than when the distribution is skewed.
- RSS estimator has less variance and MSE than the SRS estimator. The variances of both methods decrease as the sample size increases. So, the relative efficiency of RSS estimator with respect to SRS estimator is bigger than 1.
- The relative efficiency of both symmetric and skewed distribution is about the same and the relative efficiency increases as the set size increases.
- The median estimator based on RSS estimator performs better than the that from SRS.

4.6 Results about the Variance:

It is very important in descriptive statistics as well as inferential statistics to measure the dispersion of data from a center point, say the average. The variance is defined as the average of the square deviations of the data from their arithmetic mean. An unbiased estimator based on SRS is obtained when the denominator is the sample size minus 1. So, this estimator is used for both SRS and RSS in the simulation.

The variance of Rician distribution is

$$Var(x) = 2\sigma^2 + v^2 - \left(\frac{\pi\sigma^2}{2}\right) L_{1/2}^2\left(\frac{-v^2}{2\sigma^2}\right)$$

Where $L_{1/2}^2\left(\frac{-v^2}{2\sigma^2}\right)$ is the square of Laguerre polynomial. The general formula of this polynomial is given by $L_n(x) = \sum_{r=0}^{n}(-1)^r \frac{n!}{(n-r)!\,(r!)^2} x^r$.

Our aim is to compare the estimate of variance using RSS with that from SRS using simulation approach.

The simulation outcomes are given in Table 11 for skewed case and Table 12 for symmetric shape.

Table (11): Simulation outcomes to estimate the variance using RSS and SRS from Rician distribution with parameters $\sigma = 4, v = 10$.

Set size	*cycles*	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	*eff*
3	1	3.5062	0.0202	233.770	197.061	246.064	197.061	0.80
4	1	2.7626	0.0212	133.142	131.857	140.774	131.857	0.94
5	1	2.2997	0.0231	88.5512	97.9807	93.8400	97.9812	1.04
6	1	2.0154	0.0341	64.1430	79.0144	68.2047	79.0156	1.16
3	2	1.3808	0.0330	80.1481	77.9124	82.0546	77.9135	0.95
4	2	1.1743	0.0203	50.9915	55.3174	52.3706	55.3179	1.06
5	2	1.0116	0.0118	35.6182	43.1312	36.6415	43.1314	1.18
6	2	0.9067	0.0295	26.6505	35.6872	27.4727	35.6880	1.30
3	3	0.8491	0.0098	47.7911	48.6957	48.5121	48.6958	1.00
4	3	0.7611	0.0241	31.2986	35.6832	31.8779	35.6838	1.12
5	3	0.6341	0.0276	22.4117	27.7845	22.8138	27.7853	1.22
6	3	0.5857	0.0075	17.0107	22.9343	17.3538	22.9343	1.32

Table (12): Simulation outcomes to estimate the variance using RSS and SRS from Rician distribution with parameters $\sigma = 1, \nu = 14.5$ (symmetric case)

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.2410	0.0003	1.1815	1.0026	1.2396	1.0026	0.81
4	1	0.1922	0.0024	0.6814	0.6577	0.7183	0.6577	0.92
5	1	0.1545	0.0010	0.4452	0.5047	0.4691	0.5047	1.08
6	1	0.1353	0.0009	0.3307	0.3943	0.3490	0.3943	1.13
3	2	0.0954	0.0017	0.4052	0.3996	0.4143	0.3996	0.96
4	2	0.0780	0.0002	0.2626	0.2815	0.2687	0.2815	1.05
5	2	0.0721	0.0012	0.1862	0.2209	0.1914	0.2209	1.15
6	2	0.0647	0.0011	0.1396	0.1805	0.1438	0.1805	1.25
3	3	0.0586	0.0021	0.2476	0.2523	0.2510	0.2523	1.01
4	3	0.0521	0.0009	0.1609	0.1829	0.1637	0.1829	1.12
5	3	0.0456	0.0009	0.1164	0.1419	0.1185	0.1419	1.20
6	3	0.0415	0.0002	0.0881	0.1165	0.0898	0.1165	1.30

Table 11 and Table 12 show the bias, variance, MSE as well as the relative efficiency of the estimators of population variance based on SRS and RSS. We observe the followings:

- In case of skewed distribution, the bias of the RSS estimator is high compared to the estimator from SRS. However, the biasness becomes smaller as sample size becomes bigger. The amount of bias when the distribution is symmetric is smaller than for skewed case.
- The variance and MSE of the estimators become smaller as the sample size gets bigger. For large sample size, the RSS estimator has smaller MSE.
- The relative efficiency of RSS estimator increases as the set size increases and becomes bigger than 1 when the set size is relatively large. Both symmetric and skewed distributions provide estimators of about the same relative efficiency.

4.7 Results about the Coefficient of Variation:

Coefficient of variation is a measure of dispersion that's equal to the ratio of the standard deviation to the arithmetic mean. It summarizes the amount of variation as a percentage or proportion of the total. As it is free of unit, it can be used to compare groups of different units or different means.

The simulation was carried out and summarized on Table 13 with skewed case while Table 14 for symmetry case.

Table (13): Simulation outcomes to estimate the coefficient of variation using RSS and SRS from Rician distribution with parameters $\sigma = 4, v = 10$

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.0114	0.0273	0.0295	0.0315	0.0296	0.0323	1.09
4	1	0.0141	0.0171	0.0176	0.0219	0.0178	0.0222	1.25
5	1	0.0145	0.0135	0.0118	0.0166	0.0120	0.0167	1.39
6	1	0.0142	0.0101	0.0086	0.0134	0.0088	0.0135	1.53
3	2	0.0045	0.0106	0.0116	0.0133	0.0116	0.0134	1.16
4	2	0.0062	0.0073	0.0074	0.0095	0.0074	0.0096	1.29
5	2	0.0064	0.0053	0.0052	0.0074	0.0052	0.0075	1.43
6	2	0.0065	0.0039	0.0038	0.0061	0.0039	0.0061	1.58
3	3	0.0030	0.0063	0.0072	0.0084	0.0072	0.0084	1.17
4	3	0.0044	0.0042	0.0047	0.0061	0.0047	0.0062	1.31
5	3	0.0038	0.0037	0.0033	0.0048	0.0033	0.0048	1.45
6	3	0.0042	0.0031	0.0025	0.0039	0.0025	0.0040	1.57

Table (14): Simulation outcomes to estimate the coefficient of variation using RSS and SRS from Rician distribution with parameters $\sigma = 1, v = 14.5$ (symmetric case)

Set size	*cycles*	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	*eff*
3	1	0.0007	0.0077	0.0011	0.0010	0.0011	0.0011	1.03
4	1	0.0019	0.0054	0.0007	0.0007	0.0007	0.0007	1.14
5	1	0.0021	0.0041	0.0004	0.0006	0.0004	0.0006	1.29
6	1	0.0022	0.0033	0.0003	0.0004	0.0003	0.0005	1.35
3	2	0.0002	0.0033	0.0004	0.0005	0.0004	0.0005	1.09
4	2	0.0006	0.0023	0.0003	0.0003	0.0003	0.0003	1.17
5	2	0.0010	0.0019	0.0002	0.0003	0.0002	0.0003	1.28
6	2	0.0011	0.0016	0.0002	0.0002	0.0002	0.0002	1.39
3	3	0.0001	0.0020	0.0003	0.0003	0.0003	0.0003	1.08
4	3	0.0005	0.0015	0.0002	0.0002	0.0002	0.0002	1.21
5	3	0.0006	0.0012	0.0001	0.0002	0.0001	0.0002	1.28
6	3	0.0007	0.0010	0.0001	0.0001	0.0001	0.0001	1.39

From Table 13 and Table 14 show the following findings:

- The estimators are biased and the amount of bias decreases as the sample size increases. Also, their bias when the distribution is symmetric is less than for skewed distribution.

- The variances and MSEs of the estimators decrease as the sample size increases. They are less when the distribution is symmetric than when the distribution is skewed. The variance and the MSE of RSS estimator are less than that for SRS.
- RSS estimator is more efficient than SRS estimator and the relative efficiency increases as the set size increases. The efficiencies of RSS estimator when the distribution is symmetric is less than when it is skewed.

4.8 Results about the Mean Deviation:

Several measurements are available to measure the scattering of a set of data and mean deviation is one of these measurements. It is the average of the absolute deviations of the data from their average.

The properties of RSS estimator of the mean deviation were studied using computer simulation. The results were demonstrated in Table 15 for asymmetric case and in Table 16 for symmetric case.

Table (15): Simulation outcomes to estimate the mean deviation using RSS and SRS from Rician distribution with parameters $\sigma = 4, v = 10$.

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.2190	0.5671	1.6794	1.6530	1.7274	1.9746	1.14
4	1	0.1238	0.4131	1.0868	1.2475	1.1021	1.4181	1.29
5	1	0.0783	0.3297	0.7675	1.0002	0.7736	1.1089	1.43
6	1	0.0499	0.2666	0.5763	0.8393	0.5788	0.9104	1.57
3	2	0.1116	0.2733	0.8003	0.8372	0.8127	0.9119	1.12
4	2	0.0611	0.2016	0.5304	0.6277	0.5341	0.6684	1.25
5	2	0.0388	0.1576	0.3757	0.5047	0.3772	0.5296	1.40
6	2	0.0250	0.1287	0.2810	0.4259	0.2817	0.4424	1.57
3	3	0.0750	0.1783	0.5285	0.5614	0.5341	0.5932	1.11
4	3	0.0401	0.1288	0.3499	0.4273	0.3515	0.4439	1.26
5	3	0.0281	0.1088	0.2495	0.3389	0.2503	0.3508	1.40
6	3	0.0170	0.0893	0.1885	0.2844	0.1888	0.2924	1.55

Table (16): Simulation outcomes to estimate the mean deviation mean using RSS and SRS from Rician distribution with parameters $\sigma = 1, \nu = 14.5$ (symmetric case)

Set size	cycles	$Bias\ (RSS)$	$Bias\ (SRS)$	$Var\ (RSS)$	$Var\ (SRS)$	$MSE\ (RSS)$	$MSE\ (SRS)$	eff
3	1	0.0563	0.1465	0.1196	0.1175	0.1227	0.1389	1.13
4	1	0.0305	0.1076	0.0778	0.0876	0.0787	0.0992	1.26
5	1	0.0200	0.0847	0.0541	0.0714	0.0545	0.0785	1.44
6	1	0.0135	0.0696	0.0409	0.0586	0.0411	0.0635	1.54
3	2	0.0279	0.0706	0.0565	0.0592	0.0573	0.0642	1.12
4	2	0.0174	0.0512	0.0376	0.0440	0.0379	0.0467	1.23
5	2	0.0091	0.0413	0.0268	0.0359	0.0269	0.0376	1.40
6	2	0.0053	0.0343	0.0200	0.0300	0.0201	0.0312	1.55
3	3	0.0192	0.0448	0.0376	0.0401	0.0380	0.0421	1.11
4	3	0.0100	0.0340	0.0246	0.0302	0.0247	0.0313	1.27
5	3	0.0064	0.0268	0.0177	0.0238	0.0177	0.0245	1.38
6	3	0.0034	0.0224	0.0133	0.0199	0.0133	0.0204	1.53

The results of the simulation show that:

- The estimators are biased and the amount of bias decreases as the sample size increases. Also, their bias when the distribution is symmetric is less than for skewed distribution.
- The variances and MSEs of the estimators decrease as the set size increases. The variance and the MSE of RSS estimator are less than that for SRS.
- RSS estimator with respect to SRS estimator is more efficient than SRS estimator and the relative efficiency increases as the set size increases.
- The efficiencies of RSS estimator are about the same for both symmetric and skewed distribution.

4.9 Results about the Skewness:

The symmetry of a distribution is an assumption of some data analysis. Skewness is a measure of the lack of symmetry of a distribution. If one tail of the distribution is longer than the other, the distribution is skewed such as exponential distribution but if one half of the distribution is a mirror image of the other half, then it is symmetric such as normal distribution. Coefficient of skewness can be used to measure the asymmetry. One good coefficient is the ratio of the third moment around the average to the cube of the standard deviation. If the coefficient is zero, then the distribution is symmetric but if it is negative then it is negatively skewed while if it is positive then the distribution is positively skewed. See (Arnold & Groeneveld(1995), Joanes, et. al (1998))

Table (17): Simulation outcomes to estimate the skewness using RSS and SRS from Rician distribution with parameters $\sigma = 4, v = 10$.

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.1044	0.1056	0.2476	0.2506	0.2585	0.2618	1.01
4	1	0.0967	0.0972	0.3283	0.3421	0.3376	0.3516	1.04
5	1	0.0884	0.0892	0.3393	0.3706	0.3472	0.3786	1.09
6	1	0.0787	0.0825	0.3276	0.3719	0.3338	0.3787	1.13
3	2	0.0804	0.0789	0.3433	0.3739	0.3497	0.3801	1.09
4	2	0.0722	0.0672	0.3120	0.3504	0.3172	0.3549	1.12
5	2	0.0584	0.0602	0.2713	0.3160	0.2747	0.3197	1.16
6	2	0.0531	0.0563	0.2383	0.2866	0.2411	0.2898	1.20
3	3	0.0640	0.0659	0.3079	0.3303	0.3120	0.3346	1.07
4	3	0.0564	0.0558	0.2569	0.2850	0.2600	0.2881	1.11
5	3	0.0453	0.0472	0.2116	0.2478	0.2137	0.2500	1.17
6	3	0.0366	0.0413	0.1816	0.2173	0.1829	0.2190	1.20

Table (18): Simulation outcomes to estimate the skewness using RSS and SRS from Rician distribution with parameters $\sigma = 1, v = 14.5$ (symmetric case)

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	0.0018	0.0040	0.2464	0.2498	0.2464	0.2498	1.01
4	1	0.0010	0.0034	0.3289	0.3400	0.3289	0.3400	1.03
5	1	0.0018	0.0011	0.3484	0.3730	0.3484	0.3730	1.07
6	1	0.0031	0.0009	0.3398	0.3812	0.3398	0.3812	1.12
3	2	0.0007	0.0069	0.3551	0.3846	0.3551	0.3847	1.08
4	2	0.0009	0.0006	0.3330	0.3620	0.3330	0.3620	1.09
5	2	0.0003	0.0017	0.2940	0.3350	0.2940	0.3350	1.14
6	2	0.0036	0.0007	0.2620	0.3064	0.2620	0.3064	1.17
3	3	0.0005	0.0015	0.3269	0.3501	0.3269	0.3501	1.07
4	3	0.0034	0.0046	0.2757	0.3092	0.2757	0.3093	1.12
5	3	0.0009	0.0052	0.2370	0.2735	0.2370	0.2736	1.15
6	3	0.0005	0.0003	0.2036	0.2430	0.2036	0.2430	1.19

Comparing RSS to SRS when estimating the skewness using Monte Carlo simulation gave the outcomes in Table 17 and Table 18. They show the following:

- Both estimators are biased but the amount of bias decreases as the set size increases. In addition, their bias when the distribution is symmetric is less than for skewed distribution.
- The variances of the estimators decrease as the sample size increases. The variance and the MSE of RSS estimator are less than that for SRS.

RSS estimator is more efficient than SRS estimator and the relative efficiency increases as the set size increases. However, the gain in the relative efficiency is not too much

4.10 Results about the Kurtosis:

Karl-Pearson coefficient defined as the fourth moment around the mean scaled by the fourth power of the standard deviation. The distribution with kurtosis less than 3 like the uniform distribution is called platykurtic which means fewer outliers than the normal distribution. Distribution with a kurtosis bigger than 3 like Laplace distribution is called leptokurtic. See (Balanda & MacGillivary (1988) and Westfall (2014))

Our aim now is to compare the kurtosis when RSS is used to the estimator based on SRS. The properties are provided in Table 19 for skewed case and Table 20 for symmetric case.

The following comments can be provided from Table 19 and Table 20:

- The estimators are biased but the amount of bias decreases as the sample size increases. In addition, their bias when the distribution is skewed is less than for symmetric distribution.
- The variances and MSEs of the estimators decrease as the sample size increases.

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The variance and the MSE of RSS estimator are less than that for SRS.

- RSS estimator is more efficient than SRS estimator in most of the cases and the relative efficiency increases as the set size increases. However, the gain in the relative efficiency is not too much. The efficiencies of RSS estimator from both symmetric and skewed shapes are bout the same.

Table (19): Simulation outcomes to estimate the kurtosis using RSS and SRS from Rician distribution with parameters $\sigma = 4, \upsilon = 10$.

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	1.3679	1.3679	0.0000	0.0000	1.8711	1.8711	1.00
4	1	1.0676	1.0684	0.1164	0.1218	1.2562	1.2633	1.01
5	1	0.8722	0.8735	0.2252	0.2490	0.9860	1.0120	1.03
6	1	0.7352	0.7362	0.3037	0.3527	0.8442	0.8948	1.06
3	2	0.7706	0.7342	0.3243	0.3540	0.9181	0.8929	0.97
4	2	0.5957	0.5549	0.4132	0.4835	0.7680	0.7914	1.03
5	2	0.4853	0.4445	0.4383	0.5319	0.6738	0.7294	1.08
6	2	0.4073	0.3703	0.4327	0.5528	0.5986	0.6899	1.15
3	3	0.5340	0.4960	0.4694	0.5068	0.7546	0.7528	1.00
4	3	0.4098	0.3746	0.4759	0.5511	0.6438	0.6915	1.07
5	3	0.3386	0.2941	0.4446	0.5490	0.5592	0.6355	1.14
6	3	0.2817	0.2431	0.4157	0.5191	0.4950	0.5782	1.17

Table (20): Simulation outcomes to estimate the kurtosis using RSS and SRS from Rician distribution with parameters $\sigma = 1, \upsilon = 14.5$ (symmetric case)

Set size	cycles	$Bias$ (RSS)	$Bias$ (SRS)	Var (RSS)	Var (SRS)	MSE (RSS)	MSE (SRS)	eff
3	1	1.5004	1.5004	0.0000	0.0000	2.2512	2.2512	1.00
4	1	1.1992	1.2015	0.1156	0.1222	1.5536	1.5658	1.01
5	1	0.9971	0.9997	0.2273	0.2479	1.2215	1.2472	1.02
6	1	0.8542	0.8561	0.3138	0.3550	1.0435	1.0879	1.04
3	2	0.8881	0.8544	0.3308	0.3584	1.1195	1.0883	0.97
4	2	0.6936	0.6705	0.4383	0.4919	0.9195	0.9415	1.02
5	2	0.5804	0.5470	0.4709	0.5663	0.8077	0.8655	1.07
6	2	0.4954	0.4610	0.4833	0.5991	0.7287	0.8116	1.11
3	3	0.6354	0.6013	0.4940	0.5414	0.8977	0.9030	1.01
4	3	0.5039	0.4603	0.5133	0.6094	0.7672	0.8213	1.07
5	3	0.4147	0.3689	0.5018	0.6206	0.6738	0.7567	1.12
6	3	0.3574	0.3166	0.4771	0.6010	0.6048	0.7012	1.16

5. Conclusion Remarks

We studied ranked set sampling (RSS) when the samples are drawn from Rician distribution. We considered several statistics: arithmetic mean, geometric mean, harmonic mean, quadratic mean, median variance, coefficient of variation, mean deviation, skewness and kurtosis. We investigated the bias, variance and MSE of these statistics when RSS is used by using computer simulation and compared them with estimators based on SRS. The relative efficiencies of RSS estimators with respect to SRS estimators are also obtained. It is found that the estimators under the study based on RSS are more efficient than the corresponding SRS estimators. In addition, the relative efficiencies increase as the set size increases.

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