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Economic Modeling via Memory Effects Using Conformable Fractional Derivatives

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Abstract

Classical Economic models (CEM) based on integer-order classical calculus commonly assume instantaneous market clearing and memoryless agent behavior. These assumptions frequently fail to account for the widespread reality of economic hysteresis, path dependency, and delayed responses. This study suggests for the incorporation of fractional calculus into economic dynamics as a more effective framework for describing such occurrences. We concentrate on the Conformable Fractional Derivative (CFD), a contemporary fractional operator that preserves classical calculus's basic concepts, such as the product, quotient, and chain rules, while adding an adjustable memory parameter. This characteristic makes it ideal for economic modeling. We reformulate foundational economic models, counting capital accumulation and price alteration components, utilizing CFD. An expository arrangement to a fragmentary development show illustrates how a unitary subordinate arrange creates trends characterized by introductory dormancy and quickening development, adjusting more closely with watched financial behavior than its classical partner. The CFD is displayed as an effective, however numerically tractable, device for upgrading the authenticity and prescient control of financial hypothesis.

Keywords: Conformable Fractional Derivative, Economic Dynamics, Memory Effects, Path Dependence, Capital Accumulation, Price Adjustment.

1. Introduction

The inherent dynamism of economic systems is profoundly influenced by history. Consumer confidence, investment decisions, and price levels do not adjust instantaneously to new equilibria; rather, they evolve through processes shaped by past experiences and accumulated states. Traditional dynamic models, based on ordinary differential equations, are fundamentally limited in their ability to represent this "memory." They depict a world where the rate of change at any moment depends solely on the present state, ignoring the lingering effects of the past, a simplification that often leads to a significant gap between theoretical predictions and empirical observations.

Fractional calculus, the branch of mathematics dealing with derivatives and integrals of non-integer orders, provides a natural language for describing systems with memory and hereditary properties. Early applications in economics primarily utilized established fractional operators like the Riemann-Liouville and Caputo derivatives. However, these operators involve complex non-local kernels and present significant challenges in both analytical treatment and economic interpretation.

The recent introduction of the Conformable Fractional Derivative (CFD) offers a compelling alternative. Defined through a limit-based approach that generalizes the classical derivative, the CFD maintains a structurally consistent and intuitive form. Its principal advantage lies in preserving the familiar operational rules of standard calculus, thereby allowing economists to extend existing models to fractional orders with minimal conceptual overhead. This paper explores the application of CFD to economic dynamics, arguing that they provide a more nuanced and accurate representation of real-world economic behavior.

1.1. Statement of the Problem

The main problem addressed in this research is the insufficiency of classical integer-order differential equations in accounting for lasting memory effects and path-dependent behaviors found in real-world economies. Standard models, such the Solow-Swan growth model or Walrasian price adjustment mechanisms, assume frictionless and instantaneous adjustments. This produces dynamic trajectories such as exponential growth or instantaneous market clearing, which are frequently unrealistic and fail to explain phenomena such as economic inertia, sticky pricing, sluggish shock recovery, and the long-term effect of historical events on economic outcomes. There is an

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obvious need for a more adaptable and realistic mathematical framework that can easily integrate a "memory index" into economic dynamics without adding excessive mathematical complexity.

1.2. Limitations of the Study

This study is primarily theoretical and conceptual. While it provides analytical solutions and demonstrates the potential of the CFD framework, it does not include empirical calibration or testing of the models against real-world data. The estimation of the memory parameter (α) for specific economic contexts is identified as a crucial area for future research but is beyond the scope of this paper. Furthermore, the discussion is confined to relatively simple economic models (capital accumulation and price adjustment) to illustrate the core concepts, leaving the application to more complex, multi-sectoral, or stochastic models for further exploration.

1.3. Methodology

The methodology of this paper is analytical and deductive. It employs the mathematical framework of the Conformable Fractional Derivative (CFD) as defined by Khalil et al. (2014). The core approach involves:

1. **Theoretical Foundation:** Establishing the key properties of the CFD, including its linearity, product rule, quotient rule, chain rule, and its relationship to the classical derivative.
2. **Model Reformulation:** Systematically replacing the integer-order time derivatives in canonical economic models (the Solow-Swan capital accumulation model and a Walrasian price adjustment model) with conformable fractional derivatives.
3. **Analytical Solution:** Deriving and analyzing the closed-form solution for a simplified fractional economic growth model to illustrate the dynamic implications of the fractional order.
4. **Comparative Analysis:** Contrasting the dynamic trajectories generated by the fractional models with their classical counterparts to highlight the emergence of memory-driven behaviors like inertia and gradual adjustment.

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1.4. Literature Review

The application of fractional calculus in economics has been gaining traction as researchers seek to model long-range dependence and non-Markovian processes. Early work in this field, as reviewed by [4], utilized traditional fractional operators like the Riemann-Liouville and Caputo derivatives. While powerful, these operators are defined by non-local integrals with singular kernels, making them difficult to manipulate and interpret economically [3].

The seminal work of Khalil et al. (2014b) introduced the Conformable Fractional Derivative (CFD), which is defined via a direct limit that generalizes the classical derivative. Its principal advantage is the preservation of classical calculus rules, a feature that has been exploited in various scientific fields for solving fractional differential equations (Syouri et al., 2020a, 2020b). This paper builds directly upon this development, arguing that the CFD′s mathematical tractability makes it particularly suitable for economic applications, where intuition and analytical clarity are paramount. It extends the foundational economic models of Solow (1956) and others into the fractional domain using this new operator, an approach that distinguishes it from earlier efforts relying on more complex fractional calculus.

2. Theoretical Foundation: The Conformable Fractional Derivative (CFD)

This section outlines the definition and key properties of the Conformable Fractional Derivative (CFD), establishing the mathematical basis for our economic models.

Definition 2.1: [1] Conformable Fractional Derivative (CFD).

Let f: [0, ∞) →ℝ, $x > 0$, then, the Conformable Fractional Derivative (CFD) of f of order α is defined by:

$$D^{\alpha} f(x) = \lim_{\varepsilon \to \infty} \frac{f(x - \varepsilon t^{1-\alpha}) - f(x)}{\varepsilon} \text{ ,for all } x > 0, \alpha \in (0,1)$$

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If f is α-differentiable in some open interval, $(0, a)$, $a > 0$ and $\lim_{x \to 0^+} f^{\alpha}(x)$ is exist..

Then define,

$$f^{\alpha}(0) = \lim_{x \to 0^+} f^{\alpha}(x)$$

The utility of CFD in modeling in modeling stems from the adherence of key properties to classical calculus rules, as demonstrated by the following theorem:

Theorem 2.1:

Suppose that f, g are two α-differentiable functions at a point $x > 0$ when $\alpha \in (0,1)$, then,

1. $D_{CFD}\,(af + bg) = a\, D_{CFD}\,(f) + b\, D_{CFD}\,(g)\,,\ \forall\, a, b \in \mathbb{R}\,.$
2. $D_{CFD}\,(x^{\,p}) = p\, x^{\,p-\alpha}\,,\ \forall\, a, b \in \mathbb{R}\,.$
3. $D_{CFD}\,(f \times g) = f \times D_{CFD}\,(g) + g \times D_{CFD}\,(f)\,.$
4. $D_{CFD}\left(\dfrac{f}{g}\right) = \dfrac{g \times D_{CFD}(f) - f \times D_{CFD}(g)}{(g)^{2}}\,.$
5. $D_{CFD}\,(k) = 0$, for all constant function $f(k) = k$.

All branches will be proved consecutively by D_{CFD} derivation by Conformable Fractional Derivative.

Proof:

1. Applying the Definition 3.1 on $\boldsymbol{a\, f + b\, g}$ will give as,

$$D_{CFD}\,(af + bg) = \lim_{\varepsilon \to 0} \frac{af(x + \varepsilon x^{\,1-\alpha}) + bg(x + \varepsilon x^{1-\alpha}) - \big(af(x) + bg(x)\big)}{\varepsilon}$$

$$= \lim_{\varepsilon \to 0} \frac{af(x + \varepsilon x^{1-\alpha}) - af(x) + bg(x + \varepsilon x^{1-\alpha}) - bg(x)}{\varepsilon}$$

By linearity properties,

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$$D_{CFD}(af+bg) = \lim_{\varepsilon\to 0}\frac{af(x+\varepsilon x^{1-\alpha}) - af(x)}{\varepsilon} + \lim_{\varepsilon\to 0}\frac{bg(x+\varepsilon x^{1-\alpha}) - bg(x)}{\varepsilon}$$

$$= a\left(\lim_{\varepsilon\to 0}\frac{f(x+\varepsilon x^{1-\alpha}) - f(x)}{\varepsilon}\right) + b\left(\lim_{\varepsilon\to 0}\frac{g(x+\varepsilon x^{1-\alpha}) - g(x)}{\varepsilon}\right)$$

Hence,

$$D_{CFD}(af+bg) = aD_{CFD}(f) + bD_{CFD}(g). \blacksquare$$

2. Applying the Definition 2.1 on x^p gives,

$$D_{CFD}(x^p) = \lim_{\varepsilon\to 0}\frac{(x+\varepsilon x^{1-\alpha})^p - (x)^p}{\varepsilon},$$

Suppose that $h = \varepsilon x^{1-\alpha}$; therefore,

$h \to 0$, when $\varepsilon \to 0$ and $\varepsilon = (x^{\alpha-1})h$, then,

$$D_{CFD}(x^p) = \lim_{h\to 0}\frac{(x+h)^p - (x)^p}{(x^{\alpha-1})h},$$

$$= (x^{1-\alpha})\lim_{h\to 0}\frac{(x+h)^p - (x)^p}{h},$$

$$= (x^{1-\alpha})\frac{d}{dx}(x)^p,$$

$$= p\,x^{1-\alpha}x^{p-1}.$$

Hence,

$$D_{CFD}(x^p) = p\,x^{p-\alpha}. \blacksquare$$

3. Applying the Definition 2.1 on $f \times g$ produces

$$D_{CFD}(f\times g) = \lim_{\varepsilon\to 0}\frac{f(x+\varepsilon x^{1-\alpha})\times g(x+\varepsilon x^{1-\alpha}) - \left(f(x)\times g(x)\right)}{\varepsilon},$$

Add to the right side $\pm g(x+\varepsilon x^{1-\alpha}) \times f(x)$,

Then,

$D_{CFD}(f \times g) =$

$$\lim_{\varepsilon \to 0} \frac{f(x+\varepsilon x^{1-\alpha}) \times g(x+\varepsilon x^{1-\alpha}) - g(x+\varepsilon x^{1-\alpha}) \times f(x) + g(x+\varepsilon x^{1-\alpha}) \times f(x) - \big(f(x) \times g(x)\big)}{\varepsilon},$$

$$= \lim_{\varepsilon \to 0} \frac{g(x+\varepsilon x^{1-\alpha})\big(f(x+\varepsilon x^{1-\alpha}) - f(x)\big) + f(x)\big(g(x+\varepsilon x^{1-\alpha}) - g(x)\big)}{\varepsilon},$$

$$= \lim_{\varepsilon \to 0} \frac{g(x+\varepsilon x^{1-\alpha})\big(f(x+\varepsilon x^{1-\alpha}) - f(x)\big)}{\varepsilon} + \lim_{\varepsilon \to 0} \frac{f(x)\big(g(x+\varepsilon x^{1-\alpha}) - g(x)\big)}{\varepsilon},$$

$$= \lim_{\varepsilon \to 0} g(x+\varepsilon x^{1-\alpha}) \times \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon x^{1-\alpha}) - f(x)}{\varepsilon} + \lim_{\varepsilon \to 0} f(x) \times \lim_{\varepsilon \to 0} \frac{g(x+\varepsilon x^{1-\alpha}) - g(x)}{\varepsilon},$$

Since $f(x)$ and $g(x)$ are tow continues functions, then,

$\lim_{\varepsilon \to 0} g(x+\varepsilon x^{1-\alpha}) = g(x)$ and $\lim_{\varepsilon \to 0} f(x) = f(x)$,

Also,

$$f^{\alpha}(x) = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon x^{1-\alpha}) - f(x)}{\varepsilon},$$

And

$$g^{\alpha}(x) = \lim_{\varepsilon \to 0} \frac{g(x+\varepsilon x^{1-\alpha}) - g(x)}{\varepsilon},$$

Then,

$$D_{CFD}(f \times g) = f \times D_{CFD}(g) + g \times D_{CFD}(f). \qquad \blacksquare$$

Applying the Definition 2.1 on $\left(\frac{f}{g}(x)\right)$ will give,

$$D_{CFD}\left(\frac{f}{g}\right)(x) = \lim_{\varepsilon \to 0} \frac{\frac{f}{g}(x+\varepsilon x^{1-\alpha}) - \frac{f}{g}(x)}{\varepsilon}$$

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$$= \lim_{\varepsilon\to 0}\left(\frac{\frac{f(x+\varepsilon x^{1-\alpha})}{g(x+\varepsilon x^{1-\alpha})}-\frac{f(x)}{g(x)}}{\varepsilon}\right),$$

$$= \lim_{\varepsilon\to 0}\left(\frac{f(x+\varepsilon x^{1-\alpha})g(x)-f(x)\,g(x+\varepsilon x^{1-\alpha})}{\varepsilon\times g(x)\times g(x+\varepsilon x^{1-\alpha})}\right),$$

$$= \lim_{\varepsilon\to 0}\frac{1}{g(x)\times g(x+\varepsilon x^{1-\alpha})}\times \lim_{\varepsilon\to 0}\left(\frac{f(x+\varepsilon x^{1-\alpha})g(x)-f(x)g(x+\varepsilon x^{1-\alpha})}{\varepsilon}\right),$$

$\lim_{\varepsilon\to 0} g(x+\varepsilon x^{1-\alpha}) = g(x)$, for $g(x)$ is a continues function, then,

$$D_{CFD}\left(\frac{f}{g}\right)(x) = \lim_{\varepsilon\to 0}\frac{1}{g(x)\times g(x)}\times \lim_{\varepsilon\to 0}\frac{f(x+\varepsilon x^{1-\alpha})g(x)-f(x)\,g(x+\varepsilon x^{1-\alpha})}{\varepsilon},$$

Add to the right side $\pm g(x)\times f(x)$.

$$D_{CFD}\left(\frac{f}{g}\right)(x) =$$

$$\frac{1}{\left(g(x)\right)^2}\times \lim_{\varepsilon\to 0}\frac{f(x+\varepsilon x^{1-\alpha})g(x)-g(x)f(x)-f(x)g(x+\varepsilon x^{1-\alpha})+g(x)f(x)}{\varepsilon}$$

$$= \frac{1}{\left(g(x)\right)^2}\left[g(x)\times \lim_{\varepsilon\to 0}\frac{f(x+\varepsilon x^{1-\alpha})-f(x)}{\varepsilon}-f(x)\times \lim_{\varepsilon\to 0}\frac{g(x+\varepsilon x^{1-\alpha})-g(x)}{\varepsilon}\right]$$

But

$$f^{\alpha}(x) = \lim_{\varepsilon\to 0}\frac{f(x+\varepsilon x^{1-\alpha})-f(x)}{\varepsilon}\ ,\quad \text{and}\qquad g^{\alpha}(x) = \lim_{\varepsilon\to 0}\frac{g(x+\varepsilon x^{1-\alpha})-g(x)}{\varepsilon},$$

Then

$$D_{CFD}\left(\frac{f}{g}\right)(x) = \frac{1}{\left(g(x)\right)^2}\left(g(x)\times f^{\alpha}(x) - f(x)\times g^{\alpha}(x)\right).$$

Hence,

$$D_{CFD}\left(\frac{f}{g}\right)(x) = \frac{g(x)\times D_\alpha\big(f(x)\big) - f(x)\times D_\alpha\big(g(x)\big)}{\big(g(x)\big)^2}. \qquad \blacksquare$$

4. Applying the Definition 2.1 on $f(x) = k$ gives

$$D_{CFD}\left(f(x)\right) = \lim_{\varepsilon\to 0}\frac{f(x+\varepsilon x^{1-\alpha}) - f(x)}{\varepsilon},$$

but $(x) = k$; $k \in \mathbb{R}$, then

$$f(x+\varepsilon x^{1-\alpha}) = k.$$

This implies that

$$D_{CFD}(k) = \lim_{\varepsilon\to 0}\frac{k-k}{\varepsilon},$$

$$D_{CFD}(k) = \lim_{\varepsilon\to 0}\frac{0}{\varepsilon},$$

$$D_{CFD}(k) = 0. \qquad \blacksquare$$

Based on Definition 2.1 offered by Hammad and Khalil (2014a), this study will provide a proof of Theorem 2.1[7],[8] to show that there exists a relationship between the classical derivative and the conformable fractional derivative (CFD).

For a function f that is differentiable in the classical sense, this definition simplifies to:

$$D_\alpha\big(f(x)\big) = x^{1-\alpha}\frac{d}{dx}f(x).$$

This elegant form reveals that CFD is essentially a time-weighted classical derivative. When $\alpha = 1$, the weighting vanishes, and the operator reduces to the standard derivative: $D_1\big(f(x)\big) = f'(t)$.

Not meeting the Chain Rule in fractional calculus with the definitions adopted in the field led to the search for a relationship between classical calculus and fractional calculus to achieve the Chain Rule via Theorem 3.9.

Theorem 2.2. [5], [6]

Let I and J be an open intervals in $\mathbb{R}$ with $x \in I$, which $f: I \longrightarrow J$ and $g: J \longrightarrow \mathbb{R}$ be two functions. Suppose that f is α-differentiable at x, and that g is α-differentiable at $f(x)$, then $(g \circ f)(x)$ is α-differentiable at x and

$$(g \circ f)^{\alpha}(x) = g'\big(f(x)\big) \times f^{\alpha}(x)$$

Proof:

To find $(g \circ f)^{\alpha}$, then apply Definition 2.1 on it, to get

$$(g \circ f)^{\alpha}(x) = \lim_{\varepsilon \to 0} \frac{(g \circ f)(x + \varepsilon x^{1-\alpha}) - (g \circ f)(x)}{\varepsilon}$$

$$= \lim_{\varepsilon \to 0} \frac{g(f(x + \varepsilon x^{1-\alpha}) - g\big(f(x)\big)}{\varepsilon},$$

Multiplied by

$$\frac{f(x + \varepsilon x^{1-\alpha}) - f(x)}{f(x + \varepsilon x^{1-\alpha}) - f(x)}$$

$$(g \circ f)^{\alpha}(x) = \lim_{\varepsilon \to 0} \frac{g(f(x + \varepsilon x^{1-\alpha}) - g\big(f(x)\big)}{f(x + \varepsilon x^{1-\alpha}) - f(x)} \times \frac{f(x + \varepsilon x^{1-\alpha}) - f(x)}{\varepsilon}.$$

Hence, by Definition 2.1

$$\lim_{\varepsilon \to 0} \frac{f(x + \varepsilon x^{1-\alpha}) - f(x)}{\varepsilon} = f^{\alpha}(x).$$

Now, let, $\beta = f(x + \varepsilon x^{1-\alpha}) - f(x)$ then replace $f(x + \varepsilon x^{1-\alpha}) - f(x)$ by β in,

$$\lim_{\varepsilon \to 0} \frac{g(f(x + \varepsilon x^{1-\alpha}) - g\big(f(x)\big)}{f(x + \varepsilon x^{1-\alpha}) - f(x)},$$

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When $\varepsilon \to 0$, then $\beta \to 0$,

So

$$\lim_{\varepsilon \to 0} \frac{g(f(x+\varepsilon x^{1-\alpha}) - g\big(f(x)\big)}{f(x+\varepsilon x^{1-\alpha}) - f(x)} = g'\big(f(x)\big)$$

$$(g \circ f)^{\alpha}(x) = \lim_{\beta \to 0} \frac{g(f(x)+\beta) - g\big(f(x)\big)}{\beta} \times \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon x^{1-\alpha}) - f(x)}{\varepsilon}.$$

Hence,

$$(g \circ f)^{\alpha}(x) = g'\big(f(x)\big) \times f^{\alpha}(x). \quad \blacksquare$$

These properties, illustrated by Theorem 2.1 and Theorem 2.2, facilitate the direct extension of classical economic differential equations into the fractional domain.

2.2 Economic Interpretation of the Order α:

The parameter α serves as a continuous "memory index" for the system:

- As $\alpha \to 1$, memory effects vanish, and the system behaves classically, with instantaneous adjustment.
- As $\alpha \to 1$, memory effects intensify. The system's current state becomes heavily influenced by its history, manifesting as inertia, sticky adjustments, and persistent trends.

3. Fractional Dynamics in Economic Models

We now demonstrate the application of CFD by reformulating two canonical economic models.

3.1 Capital Accumulation with Memory:

Let's look at the standard model, the "Solo-Swan model," of the equation for capital accumulation:

$$D\big(K(t)\big) = sY(t) - \partial K(t),$$

Here, $K(t)$ represents the capital stock at time t, s denotes the savings rate, $Y(t)$ is the level of output, and ∂ stands for the depreciation rate. This setup assumes that all savings are invested without delay and that capital depreciation occurs instantly.
We take the production function to follow the Cobb–Douglas form:

$$Y(t) = AK^{\beta}.$$

Hence, replacing the time derivative with a Conformable Fractional Derivative (CFD) introduces memory:

$$D_{\alpha}(K)(t) = sY(t) - \partial K(t).$$

Substituting the definition of the CFD yields:

$$t^{1-\alpha}\frac{dK}{dt} = sY(t) - \partial K(t),$$

Which can be rearranged as:

$$\frac{dK}{dt} = t^{\alpha-1}\big(sY(t) - \partial K(t)\big).$$

This new equation indicates that the effective force driving capital change net investment is scaled by a time-dependent factor $t^{\alpha-1}$. For $\alpha < 1$, this factor is large for small t and decays over time, modeling an economy where the responsiveness of capital accumulation is initially dampened (e.g., due to institutional rigidities or slow technology adoption) but increases as the system evolves.

3.2 A Fractional Price Adjustment Mechanism:

A classic Walrasian price adjustment model is:

$$\frac{dp}{dt} = \gamma\big(T(p) - S(p)\big)$$

Where $\gamma > 0$ is the speed of adjustment. This often leads to unrealistically fast convergence.

Its fractional counterpart is:

$$D_\alpha\big(p(t)\big) = \gamma\big(T(p) - S(p)\big)$$

This leads to:

$$\frac{dp}{dt} = \gamma\, t^{\alpha-1}\big(T(p) - S(p)\big).$$

Here, the adjustment speed is no longer constant but is modulated by $\gamma\, t^{\alpha-1}$. This captures the concept of "sticky prices" more naturally, as the market's responsiveness to disequilibrium is a function of time and the memory parameter α, leading to more gradual and empirically plausible price paths.

4. An Illustrative Example: Fractional Economic Growth

To illustrate the dynamical suggestions, consider a rearranged growth model where capital collects at a rate proportional to its existing stock:

$$D_\alpha\big(K(t)\big) = rK(t), \quad r > 0.$$

Using the CFD definition:

$$t^{1-\alpha}\frac{dK}{dt} = rK(t).$$

Separating variables and integrating:

$$\frac{dK}{K(t)} = rt^{\alpha-1}dt.$$

$$\int \frac{1}{K}\,dK = \int rt^{\alpha-1}dt,$$

To get

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$$\ln K(t) = \frac{r}{\alpha} t^{\alpha} + c, \qquad \text{where } c \text{ is constant}$$

Then, by take $K(0) = K_0$, we obtain:

$$K(t) = K_0 \left(e^{\left(\frac{r}{\alpha} t^{\alpha}\right)} \right).$$

4.1 Analysis of the Solution:

This result stands in contrast to the classical exponential growth $K_0 e^{(rt)}$. The fractional solution exhibits a fundamentally different growth profile:

- For $\alpha < 1$, the growth is sub-exponential. The function t^{α}grows more slowly than t for small t, modeling an initial phase of inertia or slow build-up.
- The growth accelerates over time but remains slower than the classical case in the early stages. This "slow-then-accelerating" trajectory is reminiscent of economies experiencing gradual technological diffusion or structural transformation.
- The classical model is recovered precisely when $\alpha = 1$.

5. Discussion and Implications

The application of Conformable Fractional Derivatives (CFD) offers a worldview shift in economic modeling. By basically altering the derivative order α, economists can permeate their models with a spectrum of dynamic behaviors that are otherwise troublesome to capture.

A lower value of α (strong memory) can represent economies characterized by:

- Long gestation periods for investment projects.
- Significant bureaucratic and institutional frictions.
- Persistent habits in consumption and slow-changing inflation expectations.
- Slow recovery of consumer and investor confidence following a shock.

Conversely, a higher α (weak memory) approximates the classical, frictionless ideal of highly flexible and responsive markets.

The key advantage of the CFD over other fractional operators is its operational simplicity. It allows for the direct derivation of analytical solutions and a clear economic interpretation of the fractional order, bypassing the mathematical complexities associated with traditional fractional calculus.

6. Conclusion

This paper has demonstrated that the conformable fractional derivative is a potent and elegant tool for incorporating memory and path dependence into economic models. By generalizing classical differential equations while preserving their core operational rules, the CFD provides a seamless pathway to more realistic and dynamic economic theory. The illustrative models of capital and price dynamics show how fractional-order systems naturally produce inertia and delayed adjustment, features that are ubiquitous in real economies but absent in standard models.

Future research should explore the empirical estimation of the memory parameter α for different economic sectors and time periods. Furthermore, the application of conformable derivatives holds great promise in other domains, such as modeling financial market volatility with long memory, formulating monetary policy rules that account for expectation persistence, and building more nuanced models in behavioral macro -economics.

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