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Applying (ARFIMA) Model for Forecast the Saudi Stock Market Prices

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Abstract

The interest in the topic of time series forecasting has increased during the recent years and thus appeared specific modern methods, for example Autoregressive Fractional Integrated Moving Average model (ARFIMA), or what is called long memory model, which use fractionally difference (d) instead of integer which used in ARIMA models. In this study we displayed long memory feature and its tests.

Our discussion supported by analysing the real time series (daily closing index of the Saudi Arabia Stock Market prices) over period 1/1/2018 to 19/12/2022, including 1240 observations, to make a good model to giving forecast results for future, using statistical tests and statistical software (R- 4.2.2 program).

Firstly we checked that the data series was unstable by testing unit root, using Dickey fuller and Philips Perron tests ; and confirmed the presence of a long memory pattern in it by calculating the Hurst exponent (H), then calculated the fractional differential coefficient (d), determined the appropriate models for analysis and prediction, then the best model was chosen among them based on the comparison criteria, then predicted the values for the next 5 days, R - 4.2.2 software was used in all of these tests and predictions Statistical results indicated that the optimal model to represent the data series is ARFIMA (3,0.383,2) which used to predict future values.

Keywords: Time Series Models, Forecasting, Fractionally Difference, ARFIMA models, Stock Market prices.

1. Introduction

A time series is a set of observations generated sequentially over time. (Box, Jenkins, Reinsel, & Ljung, 2016) Time series modelling is a dynamic research area which has attracted attentions of researchers' community over last few decades. The main aim of time series modelling is to carefully collect and rigorously study the past observations of a time series to develop an appropriate model which describes the inherent structure of the series. This model is then used to generate future values for the series, to make forecasts. Time series forecasting thus can be termed as the act of predicting the future by understanding the past. (Adhikari, 2008; Raicharoen, Lursinsap, and Sanguanbhokai, 2003).

The time series data is visualized and analyzed to find out mainly three things, trend, seasonality, and heteroscedasticity. (Bharatpur, 2022).

There are several models in the time series including Autoregressive Integrated Moving Average (ARIMA), Seasonal Autoregressive Integrated Moving-Average (SARIMA), Seasonal Autoregressive Integrated Moving-Average with Exogenous Regressors (SARIMAX), Vector Autoregression Moving-Average (VARMA), Vector Autoregression Moving-Average with Exogenous Regressors (VARMAX), Autoregressive Fractional Integrated Moving Average (ARFIMA). From several time series models, the Autoregressive Fractional Integrated Moving Average (ARFIMA) is a model that is able to capture extreme fluctuations and long memory by using the fractionally difference (d) instead of integer which used in ARIMA model when the dataset nonstationary, this happens if the case under study experiences a continual change over time. (Burnecki, Sikora, 2017; José, Belbuteab, Alfredo& Pereirac,2015; Kartikasari, 2020; Kartikasari, Yasin, & Maruddani, 2020;)

The aim of this study is examining the effectiveness of Autoregressive Fractional Integrated Moving Average (ARFIMA) models in modelling Saudi stock market price index and select a good model to predict the future prices. The questions which we want to answer are, how to use the ARFIMA models in time series modelling? and how to choose the best model for forecasting?

2. Literature Review

2.1. Historical Background

The ARFIMA (p,d,q) process was first introduced by Hurst in (1951),in the field of hydrology, then by Granger and Joyeux in (1980), and Hosking in (1981).The most useful feature for this process is the long memory. Long memory means the effect of a shock is permanent and affects all future values of the time series, this property is reflected by the hyperbolic decay of the autocorrelation function or by the unboundedness of the spectral density function of the process. While in an ARMA structure, the dependency between observations decays at a geometric rate, Geweke and Porter-Hudak (1983) also found ARFIMA models useful for forecasting other leading indicator series.

Several studies using this method in predicting an event in some fields such as economics, agriculture, health, hydrology and finance:

- Granger and Joyeux (1980) used a fractionally differenced model with no short-term components to model the US monthly index of consumer food prices for the period January 1947 to June 1978, based on minimizing the 10-step-ahead forecast errors, they estimated d to be approximately 0.35, after first differencing the original time series. (Granger, and Joyeux, 1980).

- Sowell (1992) applied the ARFIMA process to correctly model the trend behaviour of the postwar US real GNP data. This time series has sample size equal to 172 observations and the author compared test of hypothesis using fractional and non-fractional ARIMA processes in modelling the long-run behaviour of the series. (Sowell, 1992b).
- Sahed Abdalgader and Mkidiche Mohammed (2014): In their study, they have dealt with the oil price model using long memory models (ARFIMA) to forecast oil price during the twelve coming months starting from January until December 2014. (Abdalgader, & Mohammad, 2014).
- Shadi Eltilabani and Mohammed Elsoas (2016) investigated the use of ARFIMA model as in predicting indicator of Food and Agriculture Organization, using the data from the interval (Jan 1990 to May 2014) they determined the value of the fractional difference parameter (d=0.418), the results of their research indicated to a rise in the prices during the period June/2014 to December/2014. (Eltelbany, & Elsoase, 2016).
- Kartikasari, P., Yasin, H. & Maruddani, D. A, (2020) are used ARFIMA model to predict numbers of death cases. The results of this study prove that ARFIMA (1,0.431,0) is the best model to predict data on the addition of new cases of patients dying from COVID-19. (Kartikasari, Yasin & Maruddani, 2020).
- Saif Adnan Salmana, Emad Hazim Aboudi (2022), in their study A hybrid ARFIMA-fuzzy time series (FTS) model for forecasting daily cases of Covid-19 in Iraq; they proposed hybrid model (ARFIMA-FTS) by combining the predictions of the (ARFIMA) model of the original series with the predictions of the model (FTS) for the residual series, to forecasting daily cases of Covid-19 in Iraq , for the period from 24/2/2020 to 11/8/2021 .(Salman, & Aboudia, 2022).
- Monge & Gil-Alana (2021); and Monge, & Infante. (2022) also used ARFIMA model to predict Crude oil prices.

2.2. ARFIMA process

Given a discrete time series process y_t with autocorrelations function ρ_j at lag j. the process possesses long memory or is long-range dependent if the sum of the absolute autocorrelations was infinite (decaying to zero slowly at a hyperbolic rate). (Saber & Saleh, 2022).

$$\lim_{t\to\infty}\sum_{j=0}^{t}|\rho_j| = \infty \qquad (2.2.1)$$

*** Definition:**

Firstly, remember ARIMA (0,1,0) process, (y_t) defined by

$$\nabla y_t = (1-B)y_t = \varepsilon_t \qquad (2.2.2)$$

For any real-valued d, a fractionally differenced white noise (FDWN) process $\{y_t\}$ is defined by

$$\Delta^d y_t = (1-B)^d y^t = \varepsilon_t \qquad (2.2.3)$$

where Δ and B denotes the differencing and backshift operators respectively and the sequence $\{\varepsilon_t\}$ is a white noise process.

The differencing filter (called the long-memory filter, LMF) can be expanded as

$$(1-B)^d = 1 - dB + \frac{d(d-1)B^2}{2!} - \frac{d(d-1)(d-2)B^3}{3!} + \cdots$$
$$= \sum_{j=0}^{\infty}\pi_i B^j \qquad (2.2.4)$$

- When d < 1/2, $\{y_t\}$ is a stationary process with the infinite moving average representation ((using the notation of Granger and Joyeux))

$$y_t = \sum_{j=0}^{\infty} \varphi_j \varepsilon_{t-j} \quad , \qquad (2.2.5) \qquad \varphi_j$$

$$= \frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)} \frac{= (j+d-1)!}{j!\,(d-1)!} \qquad (2.2.6)$$

$$\varphi_j \sim \frac{j^{d-1}}{\sqrt{d}} \qquad as\ j \rightarrow \infty ,$$

- When d > -1/2, {y} is invertible with the infinite autoregressive representation.

$$\pi(B)y_t = \sum_{j=0}^{\infty} \pi_j y_{t-j} = \varepsilon_t \qquad (2.2.7)$$

$$\pi_j = \frac{-d(1-d)\ldots\ldots(j-1-d)}{j_!} = \frac{(j-1-d)!}{j!\,(-d-1)!} \qquad (2.2.8)$$

$$\pi_j = \prod_{t=1}^{j} \frac{t-1-d}{t} = \frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)} \qquad ,j = 1,2,\ldots \quad (2.2.9)$$

$$\pi_j \sim \frac{j^{-d-1}}{\Gamma(d)} \quad , \quad as \quad j \rightarrow \infty ,$$

- When $-\frac{1}{2} < d < \frac{1}{2}$ y is a stationary and invertible process.
- When $0 < d < 1/2$. The process is stationary with long-memory and is useful in modelling long-range persistence. The autocorrelations and impulse responses are all positive and decay at a slow hyperbolical rate.
- -1/2 < d < 0, The autocorrelations are all negative and decay hyperbolically, and the process has short memory and is said to be anti-persistent.

- When d=0.5 the process is non-stationary and invertible.
- When d= -0.5 the process is stationary and not invertible. (Hosking, 1981).

2.3. Test of Long Memory

2.3.1 Graphical test: Autocorrelation plot (ACF Plot):

The autocorrelation function measures the degree of correlation between neighbouring observations in a time series. if the ACT plot follows an as asymptotic hyperbolic or decay very slowly, the series has a long memory, mathematically the form of ACF in the ARFIMA model : (Ocker, 2014).

$$\rho_j = \frac{\Gamma(1-d)\Gamma(j+d)}{\Gamma(d)\Gamma(j-d+)} \qquad (2.3.1)$$

$$\rho_j \sim \frac{\Gamma(1-\mathrm{d})}{\Gamma(\mathrm{d})} j^{2d-1} \quad ; -0.5 < d < 0.5 \; ; \quad j \to \infty$$

2.3.2 Statistical tests:

*** R/S Statistic:**

One of the oldest methods is the rescaled-range, or simply R/S, statistic. This measure was firstly introduced by Hurst (1951) and then developed and refined by Mandelbro and Wallis (1968). The R/S statistic corresponds to the range of partial sums of deviations of a time series from the mean, rescaled by its standard deviation. Therefore, In order to evaluate the R/S statistic use this form (2.3.2.1)

$$Q_n = \frac{R_n}{S_n} = \frac{\max\limits_{1\le K \le t} \sum_{j=1}^{k}(y_j - \bar{y}_n) - \min\limits_{1\le K \le t} \sum_{j=1}^{k}(y_j - \bar{y}_n)}{\left[\frac{1}{n}\sum_{j=1}^{k}(y_j - \bar{y}_n)\right]^{\frac{1}{2}}} \qquad (2.3.2.1)$$

Where $\bar{y}$ is the sample mean of the data set.

S_n is standard deviation.

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The first bracketed term is the maximum of the partial sums of the first (k) deviations of (y_j) from the full-sample mean, which is non-negative. The second bracketed term is the corresponding minimum, which is non-positive. The difference of these two quantities is thus non-negative, so that $Q_n > 0$.

$$\frac{R_n}{S_n} = n^H \qquad (2.3.2.2)$$

H : is a Hurst exponent and it is always lies in this interval : $0 < H < 1$, then :

$$H \approx \frac{logQ_n}{\log n} \qquad (2.3.2.3)$$

If the H estimated value is 0.5 we can conclude that the process has short memory, if the estimate is within the (0.5,1) interval then it is a stationary process has long memory, the dependence is even stronger as H tends towards 1, and if the H parameter is between (0,0.5), the process is anti-persistent. (Bourbonnais, & Maftei, 2017).

Many authors identify a relation between the H parameter and the differencing parameter d. In case of an infinite variance process the relation is given by $H = d + \frac{1}{\alpha}$

but in the case of a finite variance process, the relation is simply $H = d + \frac{1}{2}$.

*** LO Statistic:**

LO is a methodology to estimate the H parameter was developed and implemented. However, many authors, namely Lo (1991), point out the incapacity of this statistic to distinguish between long and short memory. Therefore, and to overcome the lack of robustness of the R/S statistic, Lo Statistic is different from the previous one Q_n by its denominator, which takes into account not only the variances of individual terms but also the autocovariance weighted accordingly to differences of q .

$$\tilde{Q}_n = \frac{\max\limits_{1\le K\le n} \sum_{j=1}^{k}(y_j - \bar{y}_n) - \min\limits_{1\le K\le n} \sum_{j=1}^{k}(y_j - \bar{y}_n)}{\left[\frac{1}{n}\sum_{j=1}^{n}\left(y_j - \bar{y}_n\right)^2 + \frac{2}{n}\sum_{j=1}^{q} w(q)\left(\sum_{i=j+1}^{n}\left(y_j - \bar{y}_n\right)\left(y_{i-j} - \bar{y}_n\right)\right)\right]^{\frac{1}{2}}} \qquad (2.3.2.4)$$

Where $\bar{y}$ is the sample mean, also weighted $w_j(q)$ given by:

$$w_j(q) = 1 - \frac{j}{q-1} \quad , \quad q < n \qquad (2.3.2.5) .$$

Lo (1991) [23] proposed the following rule for q:

$$q = \left[\left(\frac{3n}{2}\right)^{1/3}\left(\frac{2\grave{p}}{1-\grave{p}}\right)^{2/3}\right] \qquad (2.3.2.6)$$

$\grave{p}$: estimation of the autocorrelation coefficient of order 1

Lo proves that under the hypothesis

$H_0 : \ xt \ \Rightarrow \ i.i.d.(0, {\sigma_x}^2)$

and for n which tends towards the infinity, the asymptotic distribution of $\tilde{Q}_n$ converges step by step towards $v_{cal} = \frac{\tilde{Q}_n}{\sqrt{n}} \sim V$ where V is the rank of a Brownian bridge, a process with independent Gaussian increases constrained to unity and for which H = ½. The density function of the random variable V in equation (2.3.2.7) (Bourbonnais, & Maftei, 2017)

$$f_{v(v)} = 1 + 2\sum_{k=1}^{\infty}(1 - 4k^2 v^2).e^{-2(k,v)^2} \qquad (2.3.2.7)$$

The calculation of H is done as above, and Lo analyzes the behaviour of $\widetilde{Q_n}$ under alternative long-term dependency. He then shows that:

$$v_{cal} = \frac{\tilde{Q}_n}{\sqrt{n}} \to \begin{cases} \infty \ldots. pour H \in (0.5,1) \\ 0\ldots pour\ H \in (0, 0.5) \end{cases}$$

H_0=the process has short memory, (h=0.5)

H_1 =the process has long memory.

We accept null hypothesis at significant 0.05, if $V \in [0.809 , 1.862]$. (Bourbonnais, & Maftei, 2017)

2.4. Estimate the Parameters of Model:

*** Maximum Likelihood method:**

Because it has many nice properties the ML has been widely used in estimation, it is a best method to estimate fractional parameter (d) .in this method (d) is estimated with ($\emptyset$ and θ)- the auto regressive and moving average parameters –at the same time.

***Semi parametric methods:**

And there are also some semi parametric methods like:

- Periodogram Regression method or (GPH method)
- Smooth Periodogram estimator or (dSperio method)

These methods estimate the fractional parameter (d), and then the other parameters are estimated by the classical method (Box, and Jenkins method).

3. An Application:

As an application, we used the daily closing index data of the Saudi Arabia Stock Market Prices (SSMP) from the period 1/1/2018 to 19/12/2022, the original time series with 1240 observations .

3.1. Descriptive Statistic of Data

Table 1. descriptive statistic of SSMP series

Variable	Obs	Mean	Std.dev	Min	Max
SSMP	1240	9305.782	1836.731	5959.69	13820.35

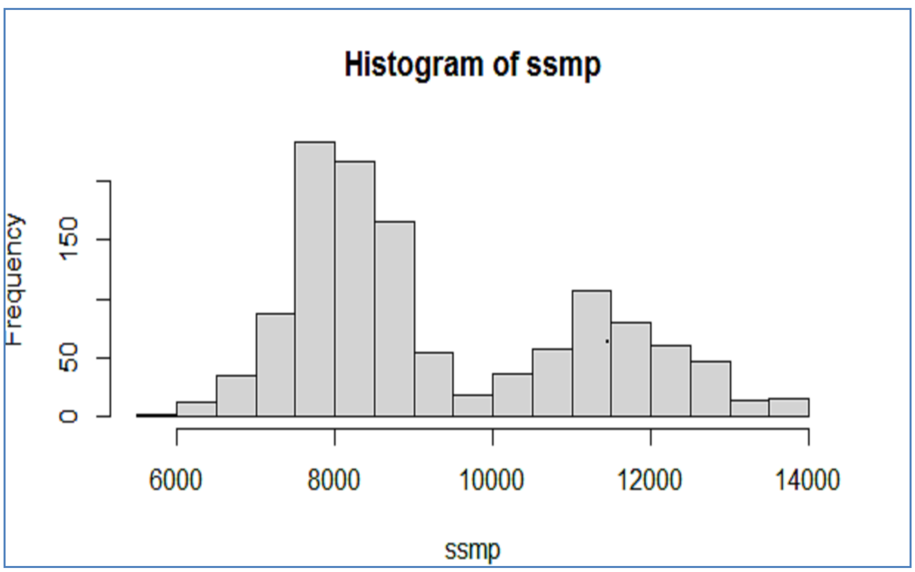

Figure 1. histogram of SSMP series, output from R- 4.2.2 program

The simple graph of series in (Figure 2), shows that the series is non-stationary, and it has trend and random fluctuations.

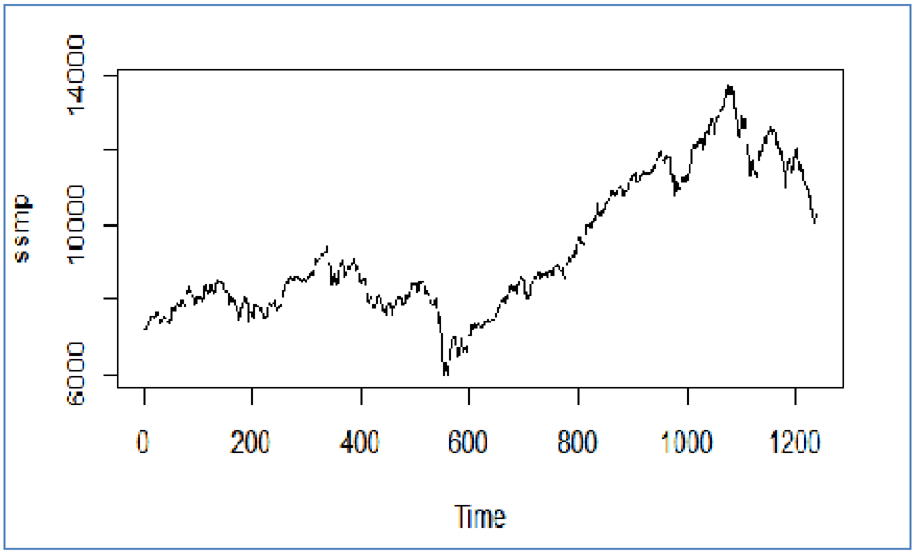

Figure 2. SSMP series plot, output from R- 4.2.2 program

3.2. Statistical Test of Stationary

The Augmented Dickey–Fuller (ADF) test (Dickey, & Fuller, 1979), and Phillips–Perron (PP) test (Phillips, & Perron,1988), they are used to stationary test , where the null hypothesis is the presence of a unit root and the alternative hypothesis is the stationary for the series, table.2 is showing the results for the tow tests , it can observed that, for both tests we cannot reject the null hypothesis at 5% of significance level, indicating that the SSMP series has a unit root.

Table 2. test of stationary for SSMP, output from R- 4.2.2 program

p- value = 0.7754	Augmented Dickey-Fuller Test
p- value = 0.8149	Phillips-Perron Unit Root Test

3.3. Long Memory Test

The next step is to identify the long memory, this is done to see whether there is a long memory effect (long-term dependency) or not. The way to do this is to observe autocorrelation function (ACF) plot in Figure3. The (ACF) decays very slowly that indicates the long memory pattern in the data.

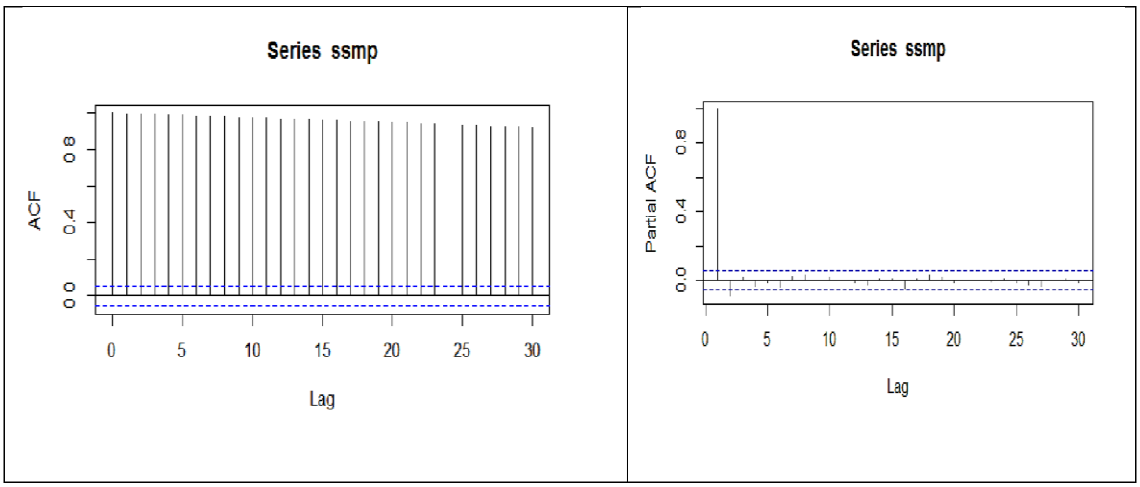

Figure 3. ACF&PACF plot for SSMP series, outputs from R- 4.2.2 program

To confirm this test, we calculate Hurst Exponent from R/S statistic by using (R-program)

$$H = 0.8835265\ ;\ \ It\ lies\ \ between\ 0.5\ and\ 1$$

that indicates to long memory process.

3.4. Estimate d- Parameter

According Hurst Exponent: $\mathrm{d} = h - 0.5 = 0.3835265$; by using (R- program) the maximum likelihood method estimated (d=0.4999) . GPH method and dSperio method gave (d estimator) greater than 0.5.

We take tow estimators for d. d= 0.4999 and d= 0.3835265,

After taking fractional difference, the series is be stationary , the graphs in figure4 and figure5 , and ADF , PP tests which illustrated in table3 are indicating that .

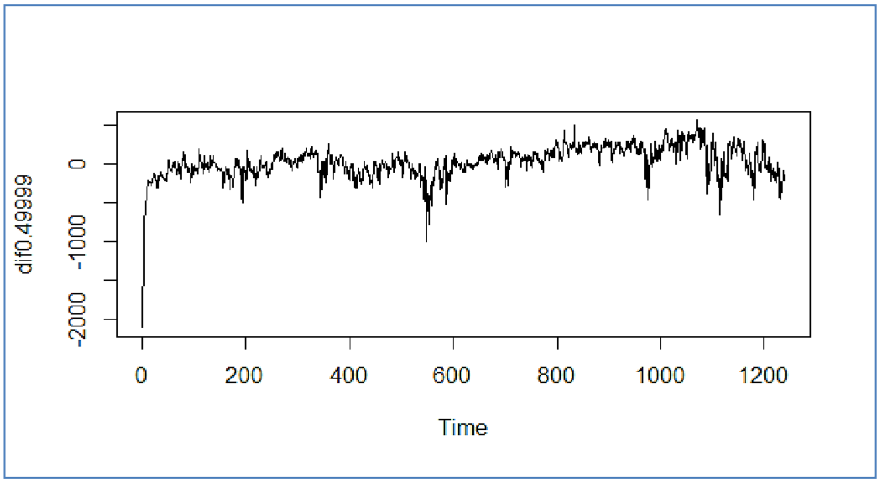

Figure 4. Time series plot after taking the fractional difference, when d=0.4999, output from R- 4.2.2 program

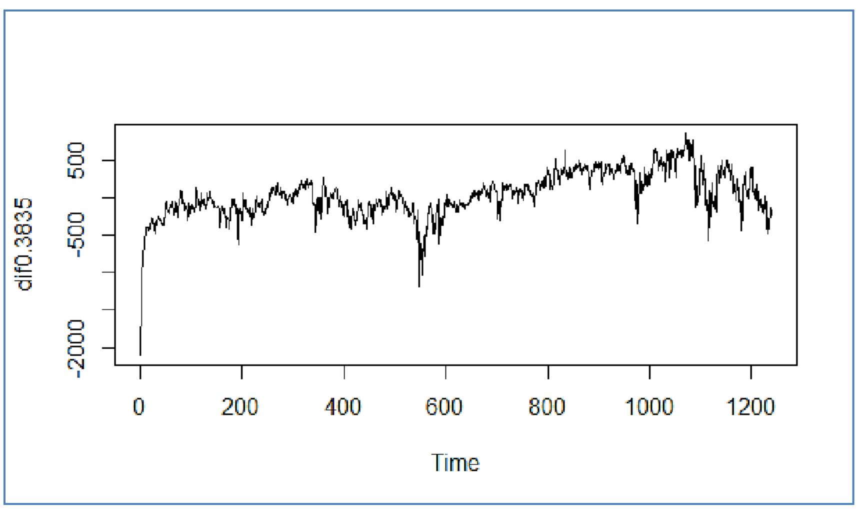

Figure 5. Time series plot after using the fractional difference, when d=0.3835, output from R-4.2.2 program.

Table 3. test of stationary after taking the fractional difference output from R-4.2.2 program

Dif0.4999 sreies	Dif0.3835 series	Test
p- value = 0.01	p- value = 0.01804	Augmented Dickey-Fuller Test
p- value = 0.01	p- value = 0.01	Phillips-Perron Unit Root Test

3.5. Model Identification and Estimation

Now we have two good estimators ford (0.376106, and 0.49967) ,in this step we shall determine the order of autoregressive and moving average (p and q) and identify models specifications , and select the best model according to :

1- Most significant coefficients.
2- Lowest volatility.
3- Highest log-likelihood statistic.
4- Lowest Akaike Information Criteria (AIC). (Akaike, 1973)

Among all the models, the models with the most significant parameters are presented in table 4 bellow.

Table 4. output from R- 4.2.2 program

Model	(p,d,q)	Sigma(eps)	Log-likelihood	AIC
Model1	(2,0.49,1)	94.6289	-7402.381	14816.4
Model2	(3,0.38,2)	94.2745	-7397.408	14808.8

It can noted that model2 has minimum AIC value and sigma value and it has highest log-likelihood value, then model2 is better , model2 parameters are presented in table 5.

Table 5. model2 parameters. output from R- 4.2.2 program

ARFIMA	ar1	ar2	ar3	ma1	ma2
(3,0.38,2)	0.713847	0.49097	-0.21753	-0.09409	0.416499

3.6 Diagnostic Tests

After getting the best ARFIMA model, the next step is checking the diagnosis, testing the residuals to see whether the residuals series achieve the assumption of a white noise, and it has a normal distribution.

A white noise test is performed on the residual after fitting the ARFIMA (3, 0.38, 2) model. (Ljung –Box) test is performed (Box, & Price, 1970), it gave (p value=0.1207) greater than 0.05, that the residuals series haven't correlation, and The autocorrelation and partial autocorrelation function graphs of the residual series are shown in Figure 6. It can be seen that the residual series is a white noise, indicating that the model is valid, although it was not passing the normality test.

Table 6. ARFIMA Model Residual, White Noise Testing, output from R 4.2.2 program.

Box-pierc test
data: residuals(model2)
X-squared = 21.6, df=15, p-value=0.1207
Nill hypotheses: uncorrelated

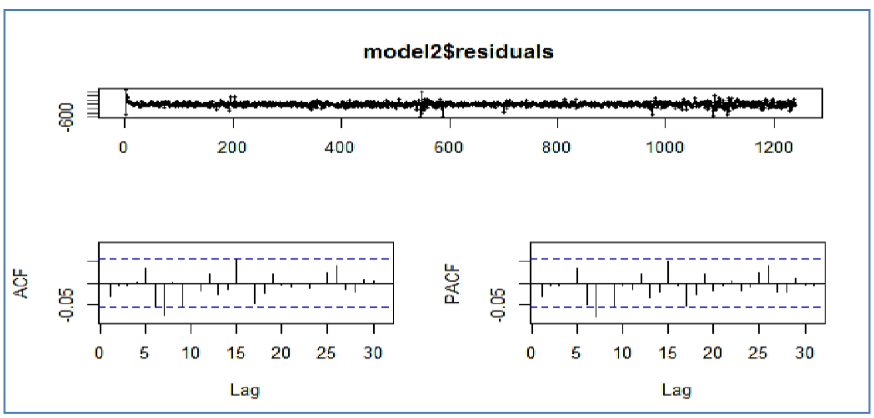

Figure 6. Autocorrelation and partial autocorrelation function graphs of the residuals series output from R 4.2.2 program

3.7. Forecast:

The results from ARFIMA model (3,0.383,2) yielded forecasting for the next 5 periods presented in table 7, figure 7 illustrated the predictive values were consistent with the original values for the series.

Table 7. Forecast results for ARFIMA (3,.383,2) model, output from R 4.2.2 program.

Date	Forecast value	Lo 80	Hi 80	Lo 95	Hi 95
20/12/2022	10167.77	10041.151	10294.39	9974.124	10361.41
21/12/2022	10152.10	9955.180	10349.02	9850.936	10453.27
22/12/2022	10137.47	9886.738	10388.21	9754.006	10520.94
25/12/2022	10122.91	9821.702	10424.12	9662.252	10583.57
26/12/2022	10108.97	9761.312	10456.64	9577.270	10640.68

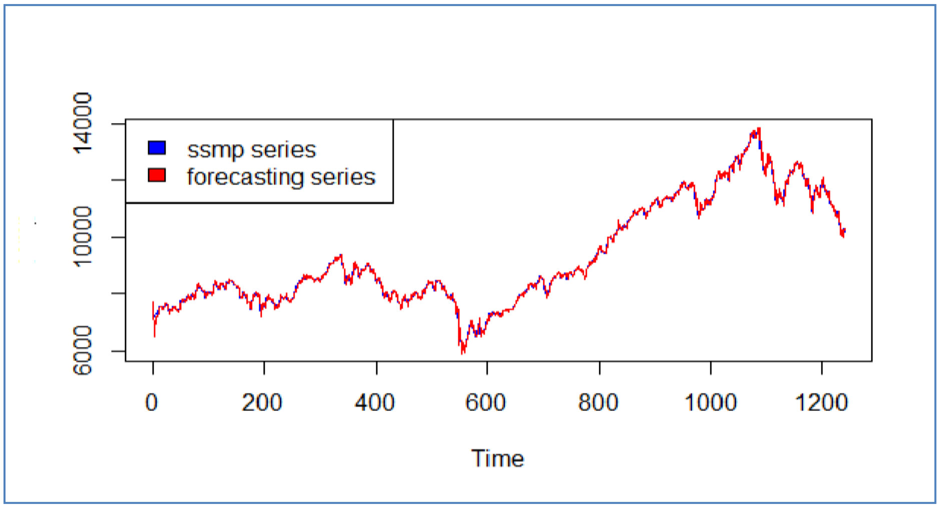

Figure 7. forecasts series from ARFIMA (3,.38,2) model, and SSMP series, output from R 4.2.2 program

4. Conclusions

In this paper has been used analysis for time series of daily closing index for Saudi Stock Market Price form (1.1.2018 to 19.12.2022) to make a good model for prediction, The result showed that the series data is non-stationary, and it has long memory; comparing the results of tow long memory models, ARFIMA (2,0.49,1) and ARFIMA (3,0.383,2), to select a good model according to some criteria's, AIC value and sigma value and log-likelihood value.

The study has been shown that ARFIMA (3,0.383,2) model is the most appropriate model and fits for future prediction with Saudi Stock Market Prices. The predictive values were consistent with the original values for the series which indicate model is efficient.

The series data was taken from website: https://sa.investing.com/indices/tasi-historical-data.

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