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Using Commutative Rings and Graphing Harmonic in Solutions and Approximation of Algebraic Equations

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Abstract

Algebraic equations are the cornerstone of many life and scientific applications, whether engineering applications, industrial applications, or environmental applications. When it comes to complex algebraic equations, solutions must be found for this complexity, and through this study, which aims to solve the problems of complex algebraic equations using the methods of reciprocal rings and drawing. Graphics and combinatorial solutions and the combination of these methods, as the two branches of algebra and geometry, or what is called geometric algebra, are combined in a flexible and innovative way through which complex algebraic equations can be solved with high accuracy and through a methodology that relied on description, analysis and comparison methodologies. Three methods were combined using machine learning and neural network techniques and obtaining on the results of solving an algebraic equation related to solving a life problem, which is arranging study courses for a number of students, where the students' requirements must be fulfilled by attending the largest number of courses without conflicting dates. The results indicated that the combination of these methods has provided many, many effective insights and strategies for solving some of the equations. As indicated by the results, the results of the model were the best by a rate ranging from 5% to 6%, and the accuracy of the model was the highest by a rate ranging between 5% to 11%, and the F1 of the model - the result also obtained the best results compared to the other two methods. At a rate ranging from 2% to 5, there are no significant differences between the expected results and the actual results of the model, which means the model is successful.

Keywords: Complexity, Reciprocal Rings, Harmonic Methods, Merger, Algebraic Equations, Equation Solving, Accuracy, Algorithms.

1. Introduction

Algebraic equations are the basis for many mathematical, scientific, and engineering applications, and there are various methods and techniques for solving them, ranging from simple to complex (Rabier, P. J., & Rheinboldt, W. C. (2002). The theory of commutative rings and graphs are among the most important theories and advanced mathematical tools through which a wide range of mathematical issues and problems can be solved, including algebraic equations. This theory combines the branches of algebra and geometry in an innovative and flexible way. This method has provided many, many effective insights and strategies to solve some Complex equations (Anderson, D. F., & Lewis, E. F. (2016).

Algebraic equations are the basis of the algebra branch of mathematics, which can be defined as the science that studies quantities, shapes, and spaces, whether geometric or non-geometric shapes, and determines the relationships between them and each other. Through these relationships, which can be called equations, concepts related to logic, abstract ideas, and even concepts related to analytical theories and random variables can be understood. Mathematics can also be defined as the science that determines the relationship between variables, whether this relationship is a fixed relationship or a relationship of different degrees, such as the first degree, which is called a linear relationship, or the second and third degrees, and so on. It also determines the nature of the relationship between these variables and the direction of this relationship. Is it Is it a strong correlation, a moderate correlation, or a weak correlation? It also helps determine the direction of this correlation. Is it a direct correlation or an inverse correlation? (Spedicato, E. (Ed.). (2012). identifying these relationships and connections between variables, some of them and others, visions and strategies can be formulated that contribute to the development of science, applications, and life matters in our real lives. When it comes to complex equations that contain many variables, many parameters, and even random variables, which cannot be solved by traditional methods, it was necessary to develop ways and methods through which these complex equations can be solved. Among these methods is the theory of reciprocal rings and the graph. This is the theory of reciprocal rings. Which can be defined as an algebraic structure consisting of a group of elements with the operations of addition and multiplication. By combining these two properties, a group of properties can be achieved that plays a vital role in number theory and experimental algebra (Morey, S., & Villarreal, R. H. (2012). As for a graph, it is a graphical relationship through which a group of relationships can be visually represented. Through this representation, the direction and connection of relationships can be determined, including the algebraic equations in which the graph is formed the vertices (nodes) represent the objects and the edges (edges) represent the relationships between these objects (Yu, D.et, al,2022). By integrating the theory of reciprocal rings, graphs, and combinatorial solutions, an integrated vision can be formed. Through this vision, we can reach the solution of the complex problems that face us while solving algebraic equations. As for the solutions Combinatorial is a set of solutions through which complex algebraic equations can be solved, such as the trial-and-error method, the algebraic analysis experiment, and the algorithmic method.

This study aims to clarify the role of the theories of commutative rings, graphs, and combinatorial solutions in solving the problems of algebraic equations, and to explain how to combine these three methods with each other to form integrated visions for solving the problems of complex algebraic equations. The study also aims to clarify the mechanism of integration and the most important applications in which this can be used. Integrating it into our daily lives. The problem of the complexity of calculations and choosing the appropriate model for the appropriate application is considered one of the most important problems facing the study of integrating the techniques of graph theory, commutative ring theory, and combinatorial solutions in solving algebraic equations (Kamche, H. T., & Kalachi, H. T. (2024).

In conclusion, the theory of commutative rings, graphs, and compromise solutions are powerful tools through which problems of complex algebraic equations can be solved, and by combining them, one can benefit from the advantages of each method separately and combine these advantages into one method. But it must be considered that the integration method is compatible with the nature of the application being used in order to obtain the greatest benefit.

2. Literature Review of Basic Concepts and Analytical Theories

Through a literary review of the basic concepts and analytical theories, it is possible to clarify and form an insightful point of view for the use of graph theory methods, commutative rings, and combinatorial solutions in solving problems of algebraic equations. In order for things to be clearer and to form a conscious understanding of the subject of the study, the most important basic concepts and analytical theory used in it must be clarified. the study.

2.1. Basic concepts:

There are some basic concepts whose meaning must be understood and their meaning clarified in order to have a clear picture of the subject of the study.

2.1.1. The theory of reciprocal rings:

The theory of commutative rings is one of the most important branches of abstract algebra. It can be defined as an algebraic structure consisting of a set of elements with the operations of addition and multiplication through which a set of properties are obtained, the most important of which are the properties of merging and substitution and the neutral elements, whether the additive neutral or the multiplicative neutral (Epstein, N, et, al,2024). Figure (1) shows the theory of reciprocal loops.

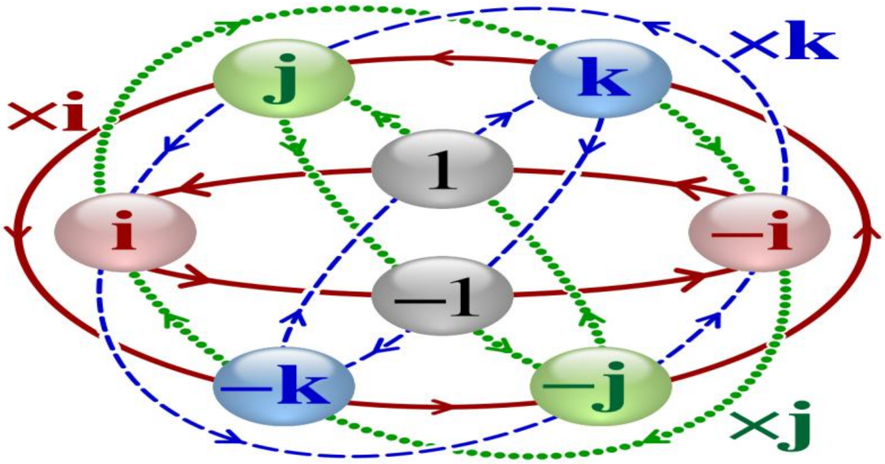

Figure (1): shows the theory of reciprocal rings (Quaternion – Wikipedia)

2.1.2. Graph theory:

Graph theory can be defined as a graphical relationship through which the relationship between variables can be clarified. It is also possible to determine the direction of this relationship and the extent to which variables relate to each other. Through this theory, relationships can be represented by graphs consisting of vertices (nodes) and edges (edges). Graphs are used in many fields, such as computer science, chemistry, and social sciences. Figure (2) shows an example of graph theory (Diestel, R. (2024).

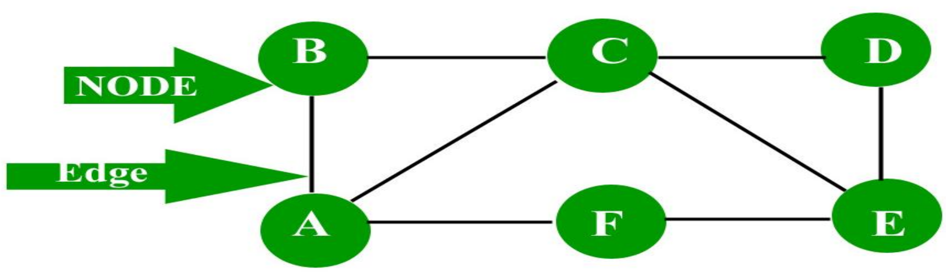

Figure (2): shows Graph theory(https://www.geeksforgeeks.org)

2.1.3. Harmonic solutions:

Harmonic solutions are a set of solutions through which problems of algebraic equations can be solved. It cannot be said that one method is better than the other, but the use of the combinatorial method depends on the nature of the equation, its degree and type, and the nature of the application in which the equation is used (Yan, Z., Dai, H., Wang, Q., & Atluri, S. N. (2023)). The most important of these combinatorial solutions are the following:

1. The trial and error method: In this method, trial, trial and error methods are used to reach appropriate results. The more attempts and repetitions there are, the more accurate the results will be.
2. Algebraic analysis: where algebraic equations can be analysed and transformed into their simplest form through which this equation can be solved.
3. Algorithms: They are a set of equations designed and programmed specifically to solve complex algebraic equation problems, especially when the parameters are unknown and when there is randomness in the variables and a lack of correlation between these variables.

2.1.4. Approximate solutions:

Approximate solutions to algebraic equations are a group of solutions that are used to solve algebraic equation problems, including the following:

1. Midpoint method: It is one of the numerical approximation methods used to find approximate solutions of functions. This method is based on dividing the possible period of the existence of the root into two halves, then evaluating the function at the midpoint and determining a new period. This step is repeated until a specified accuracy is reached. Fig(3) shows how to apply the centroid method (Krasnosel'skii, Mmet, al,2023).

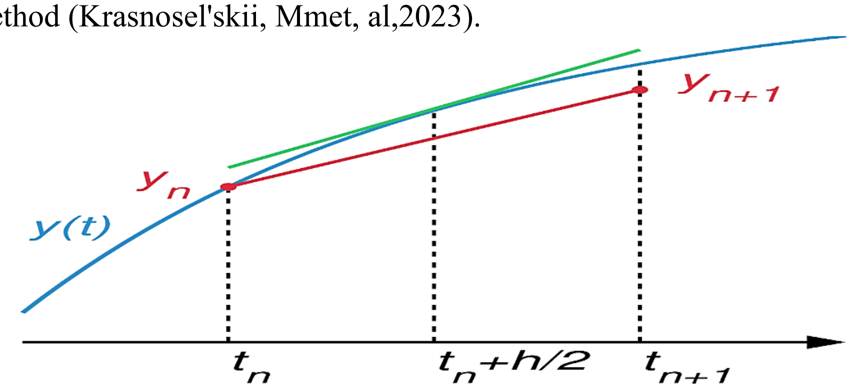

Figure (3): shows Midpoint method (Wikipedia)

2. Trapezoidal method: It is one of the harmonic numerical approximation methods and is very similar to the centroid method. It is based on dividing the potential period of the root's existence into several smaller periods, then approximating each period by a trapezoid and calculating its area under the curve. Next, the function values are calculated at the division points and the area is approximated. Then we add the total areas of the trapezoids. If the total area is close to zero, this means that the root value lies within one of the intervals. This step can be repeated to obtain higher accuracy. Figure (5) shows the trapezoidal method (Kang, H. S. (2010).

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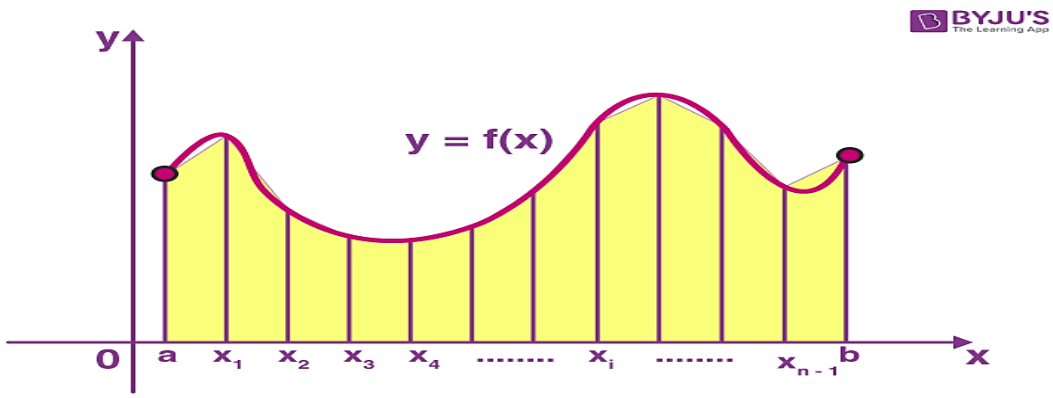

Figure (4): shows Midpoint method (BYJU'S))

3. Newton-Raphson method: It is considered one of the most famous and widely used methods, as it relies on choosing an initial estimated solution by drawing a tangent to the curve at the estimated point, and then choosing the point of intersection of this tangent with the x-axis as the starting point. This process is repeated until we reach a satisfactory solution that matches the nature of the problem and the desired goal. Figure 1 shows an example of the Newton-Raphson method (Pho, K. H. (2022).

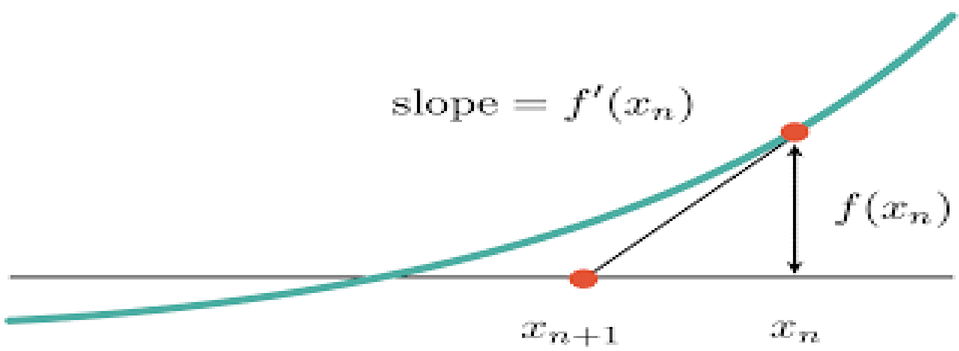

Figure (5): shows Newton-Raphson method (https://www.eigenplus.com)

4. Chord method: This method is somewhat similar to the Newton-Raphson method, but is less precise. It depends on drawing a chord connected to the curve, then finding the point of intersection of the chord with the x axis to be the starting point (Koochaksaraei, R. et.al.2017).

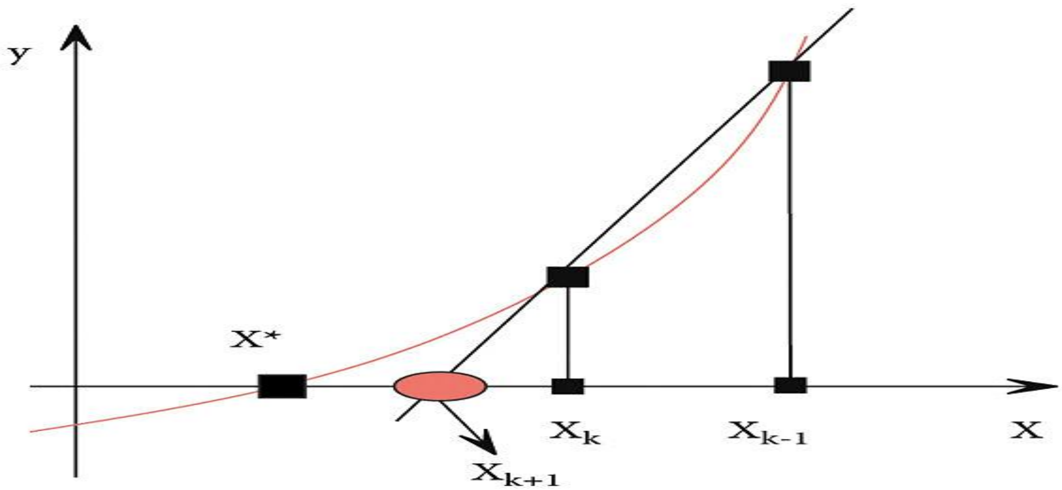

Figure (6): shows Newton-Raphson method (https://www.resesrchgate.com)

2.1.5. Algebraic equations:

An algebraic equation can be defined as a relationship between two or more variables. This relationship is represented mathematically using basic mathematical operations such as multiplication and addition. This relationship may be a direct relationship or an inverse relationship. It may be a fixed relationship, a first-degree relationship, or a relationship of multiple degrees. Where the two sides are equal. Or more precisely, both sides of the equation, for example:

$$Y = a X^{n} + b X^{(n-1)} + ... + cx + e \quad \textbf{Eq (1)}$$

where:

- A, b, c, e: Parameters
- N: degree
- X: variable

2.1.8. Geometric algebra:

Geometric algebra is a branch of mathematics, as it combines algebra tools with engineering tools to study geometric shapes in an algebraic manner (Artin, E. (2016). Where geometric shapes are described, operations on them, and algebraic analysis. As shown in fig (7).

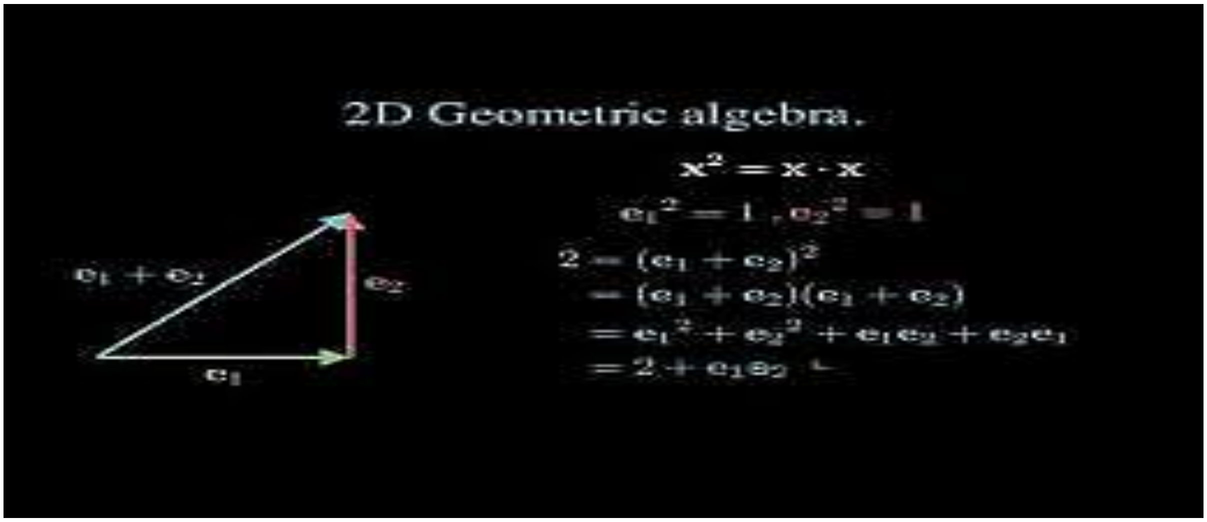

Figure (7): shows Geometric algebra((https://www.resesrchgate.com)

2.2. analytical theories:

These are the theories that will be used to explain how graph theories, commutative rings, and combinatorial solutions work, and how to use and combine them to solve algebraic equation problems.

2.2.1. Number theory and algebraic geometry:

It is the theory of the properties of numbers, such as merging, substitution, exchange, common factors, and systems for representing binary, ternary, and quadruple numbers, as well as decimal systems, fractional systems, and imaginary numbers, where numbers are placed in rings or geometric shapes that are expressed algebraically (Neukirch, J. (2013)

2.2.2. Theory Groups and algebra:

Set theory and algebra is a theory that is considered the basis for all basic concepts in modern mathematics and is applied in all fields and applications. These sets are forms that include a set of numbers and numbers that express a specific algebraic structure. Examples of these sets include the set of natural numbers, the set of integers, the set of rational numbers, and the set of rational numbers. Imaginary and set of space vectors, which were the basis for the development of mechanics (Judson, T. W. (2020).

2.2.3. Convergence theory:

The term convergence theory refers to the convergence of a set of vectors or matrices towards a specific solution, or the convergence of a series of elements or operations towards a specific element or operation. This theorem is used to compare the convergence speed in optimizing audio and visual signals using combinatorial algebraic methods and instantaneous transformations (Allen-Zhu, Z., Li, Y., & Song, Z. (2019, May).

2.2.4. Harmonic analysis theory:

Harmonic analysis theory is a theory used to analyse the combinations and combinatorial structures that link the relationship between variables in algebraic equations, as well as the algebraic equations that are related to them. Thus, it is possible to choose one of the appropriate combinatorial methods to solve algebraic equations, whether it is trial and error methods, algorithmic methods, or numerical approximation methods (Benedetto, J. J. (2020).

2.2.5. Statistical analysis theory:

Statistical analysis theories are a set of theories through which sets of data can be analysed to ascertain the variability of the data, the degree of its importance, the quality of the data, its suitability for the applications used in it, and its suitability for analysis, as well as studying the correlation between the data and linear regression and its direction. It is compatible between the data and the models and applications used in it (Bain, L. (2017).

3. The Method and Methodology

Several methodologies were adopted in the study, including the descriptive methodology in describing the basic concepts, analytical theories, quantitative and scientific methodology in collecting data from previous studies and Internet rules, and the comparative methodology to compare the results. A model was created by integrating the theory of reciprocal loops, graph theory, and combinatorial solutions, then using techniques and algorithms in MATLAB. You entered data entry and created a code whose function is to integrate the theories of commutative rings, graphs, and combinatorial solutions, and used this code to solve complex algebraic equations that cannot be solved by traditional methods, then measured the accuracy of the model in terms of performance speed, efficiency, and computational complexity. F1-score, as well as recall and random descriptors after combining three techniques using deep machine learning techniques and neural networks. The graph was used to represent the elements of each ring of the graph, and the multiplication operations between the elements were represented by edges connecting the two opposite vertices, then many of the properties of the ring were extracted from the graph. The graph, such as the presence of neutral elements and interchangeable elements and vice versa, then searching for solutions using the properties of the graph and solutions using combinatorial solutions, comparing those solutions and choosing the best and most optimal solution, where graphical messengers were used to search for compatible solutions using reciprocal loops(Tirelli, M, et, al,2019).

3.1. The applied framework of the study:

In this part, we will present the practical steps and stages of applying the study through an applied framework for the steps and stages, as shown in Figure No. (8), as well as a flute chart for the steps of the study, as shown in Figure No. (9). Scientific and academic methods and theories have been considered in applying the steps of the study in accordance with previous studies. And the principles of scientific research.

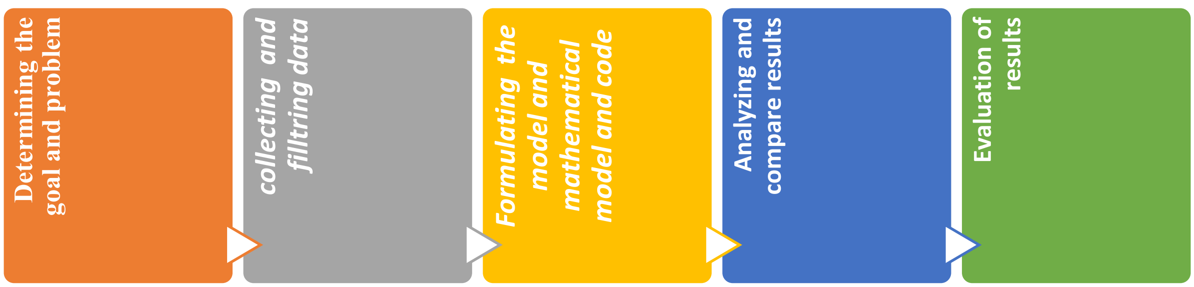

Figure (8): shows the applied framework of the study. (by author).

The applied framework of the study explains the stages of the study, starting with defining the problem and formulating the goal, then collecting data based on the sources of previous studies and Internet databases, then formulating the model by integrating graph theory, commutative rings, and combinatorial solutions to solve the problems of complex algebraic equations related to the subject of providing educational and study courses in One of the educational institutions that offers a wide range of courses. What is required is to meet the different requirements of students at specific time periods and avoid conflicts in course dates. Machine learning techniques and recurrent neural networks will be used after formulating the mathematical model for the equations that express the merging graph, reciprocal rings, and combinatorial solutions. Then extract, compare and evaluate the results, then draw conclusions and make recommendations.

3.2.procedures:

1. Construct a graph where:

- Define the vertices, where each vertex represents a basic course in the chart.
- Identify the edges where courses are linked so that no time conflicts occur and any student can attend more than one course without scheduling confusion.
- Determining the weights, where each edge is weighted and expresses the extent of the importance of the course to the student, when it became famous, and how popular this course is. There are courses that have a high demand and other courses that have a weak demand.

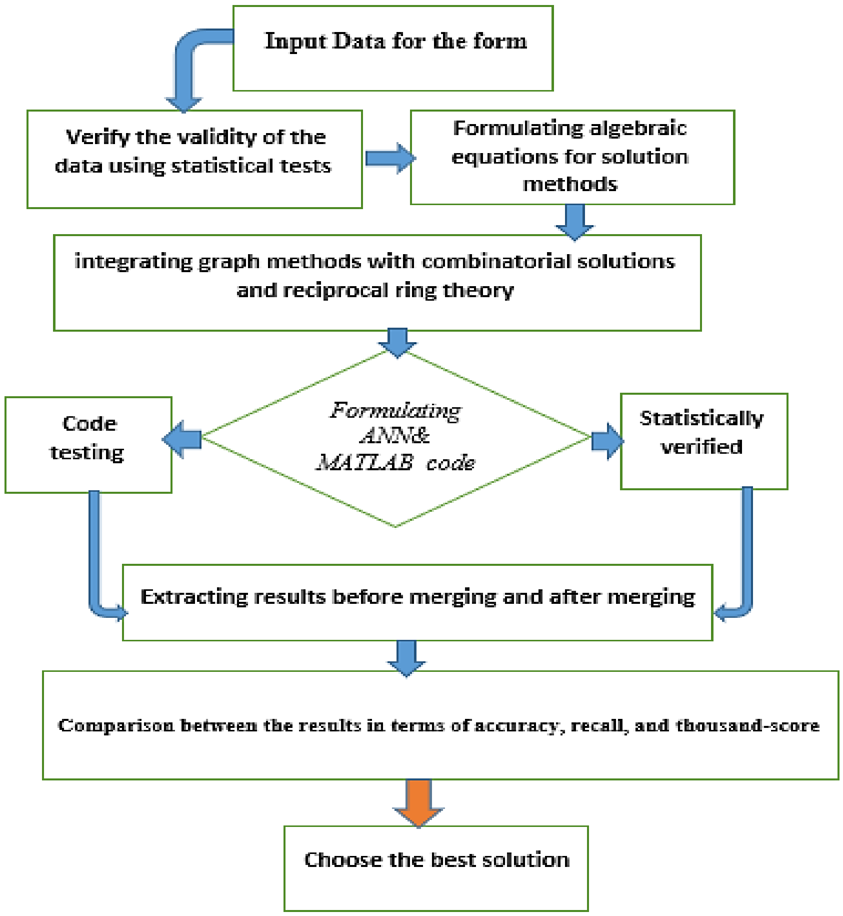

Figure (9): study flow chart(researcher)

2.Application of the reciprocal loop theory:

- Where the cycle is determined and is represented by a group of academic courses in the reciprocal loop system, where courses are combined and merged, considering time conflicts. A
- Ideals: The concept of ideals can be used to represent enrollment restrictions, such as a minimum number of credit hours or specialty requirements.

3.Finding compatible solutions:

- Then the superior solutions are determined by formulating the problem in the form of an algebraic equation in the reciprocal loop, where the unknown is the registration schedule for each student, where each student can achieve the maximum benefit from attendance in the largest number of courses, considering constraints such as time conflicts.

- Using algorithms and combinatorial solutions to achieve the optimal and best solution for each student, so that he can attend the required courses without conflict in the dates and time periods of these courses.

3.3. Formulate the mathematical model

Formulating the mathematical model Suppose we have the following:

- M: a range of courses.
- S: a group of students.
- C: a binary matrix that identifies the conflict between cycles. If there is a collision between cycles i and j, then C (i, j) = 1, otherwise C (i, j) = 0.
- P: a matrix outlining students' preferences for courses. P (i, j) represents student i's preference for course j.
- x_ij: a binary variable equal to 1 if student i is enrolled in course j, and equal to 0 otherwise.

1) Then Each student can register for a limited number of courses:

$$\sum\_j x\_ij \leq k\_i \forall i \in \sum\_\{j \in M\} x\_\{ij\} \leq k\_i \forall i \in S \quad \text{Eq (2)}$$

where:

k_i: s the maximum number of courses that student i can enroll.

2) A student cannot register for two conflicting courses:

$$x\_\{ij\} + x\_\{ik\} \leq 1 \forall i \in S, \forall j, k \in M \quad \text{Eq (3)}$$

where:

C (j, k) =1

3) Demand for the course:

$$\sum\_\{i \in S\} x\_\{ij\} \leq capacity\_j \forall j \in M \quad \text{Eq (4)}$$

where:

capacity-j: is the maximum number of students that can be enrolled in course j.

4) Then define the objective function, which is as follows:

Objective function: We aim to maximize the sum of student preferences for all registered courses. This can be represented by the following objective function:

Maximize:

$$\boldsymbol{Maximize}: \sum\_\{i \in S\} \sum\_\{j \in M\} P(i,j) * x\_\{ij\} \quad \textbf{Eq (5)}$$

Where the objective function is to maximize the sum of students' preferences for all enrolled courses.

3.4. Model formulation:

Using neural network technology, graph techniques, reciprocal loops, and combinatorial solutions will be combined to obtain the optimal solution. Figure No. (10) shows the neural network model.

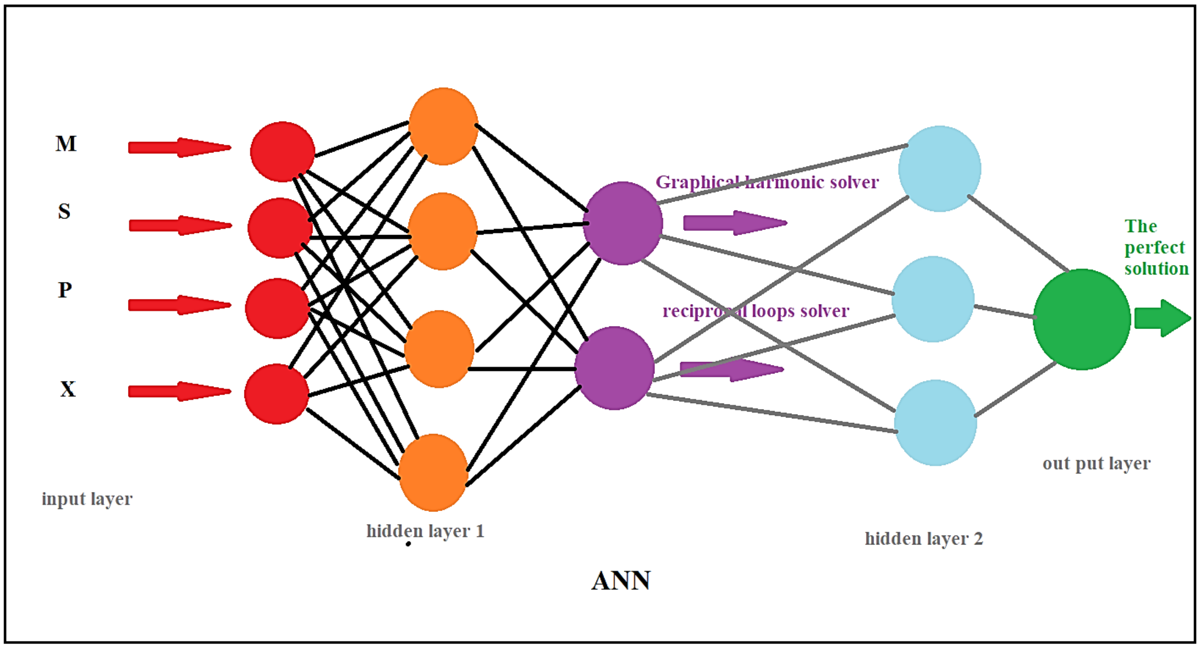

Figure (10): Integrating solutions using a neural network

3.5.Statistical analysis:

According to the data and the designed model, a set of statistical tests will be conducted on the data set, including the Anova test, which determines the importance of the data and the extent of its variation through the p-value, and its marginal value is 5%, where if the value exceeds 5%, the data is not important, and if the value is less From 5%, the data becomes more important, less than it. Also, the coefficient of variation, which determines the extent of the significance of the data and its statistical significance, whether it can be relied upon or not, and whether it fits the model or not. Through the correlation test, an equation can be formed linking the arithmetic mean and the value, the arithmetic mean of the accounting value, and the coefficient. They descended as follows:

1. Pearson correlation coefficient: It measures the strength and direction of the linear relationship between two variables. Its value ranges between -1 and 1, where a value of 1 indicates a strong positive collinearity, a value of -1 indicates a strong negative collinearity, and a value of 0 indicates no collinearity.

$$r = \Sigma\left((x_i - \bar{x})(x_j - ş)\right) / \left(\sqrt{\Sigma (x_i - \bar{x})^2} * \sqrt{\Sigma (x_j - ş)}\right) \quad \textbf{Eq (6)}$$

where:

- Y: correlation coefficient
- x_i: The value of the solution by the graphical harmonic method i
- $\bar{x}$: arithmetic mean of the graphical harmonic solution
- xj: value of the solution of the cross-links
- s: arithmetic mean Solution with alternating rings

2. Regression analysis: Simple regression model: If we assume a linear relationship between market value and accounting value, we can use a simple regression model:

$$Y = \alpha + \beta x + \varepsilon \quad \textbf{Eq (7)}$$

where:

- y: solution with alternating rings
- x: Solution using combinatorial methods and graphs.
- α: intercept
- β: slope coefficient
- ε: random error

Estimating parameters: The coefficients α and β can be estimated using the least squares method. Then solving the equations: These equations are solved to calculate the values of the parameters to be estimated.

4. Results and Discussion

This part will present the results and analyse them in terms of accuracy, flexibility, recall, thousand-one-score factor, and comparison between the results before and after the merger.

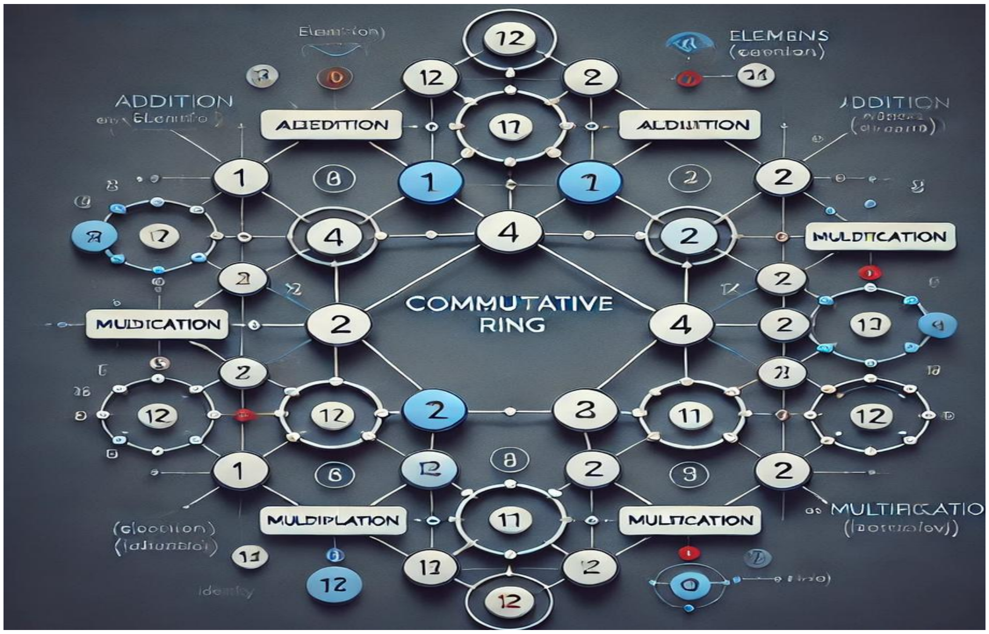

Figure (11): shows the diagram of the reciprocal loop with combinatorial solutions

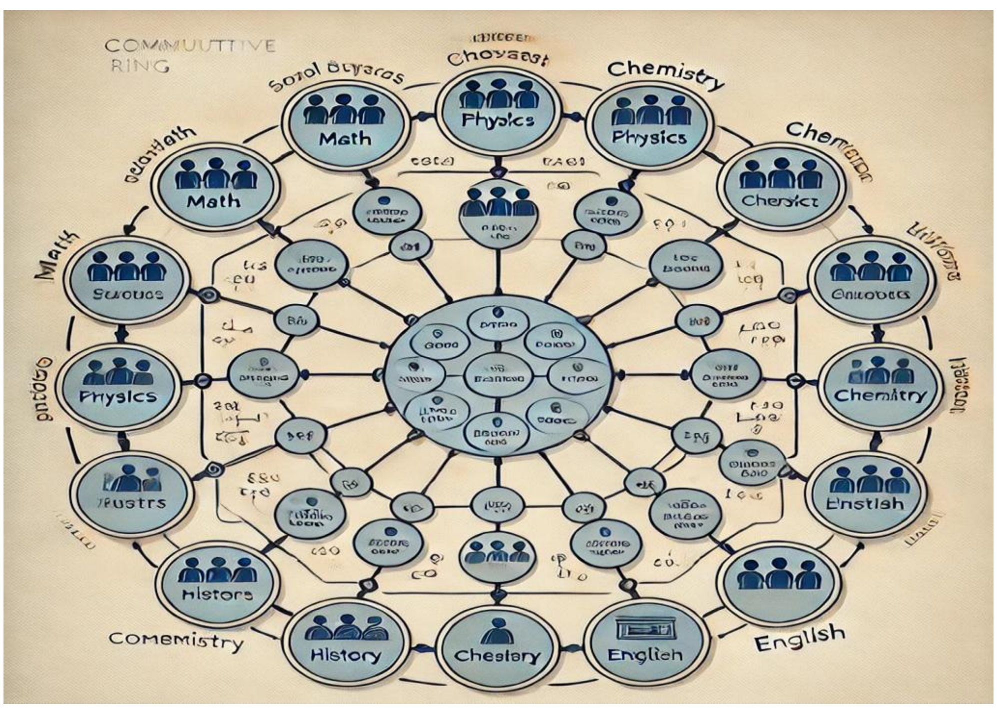

Figure (12) shows the reciprocal loop diagram of the model

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In Figure No. (11), the numbers represented the number of combinatorial solutions, and the numbers in Figure No. (12) were replaced by elements representing the academic subjects and their relationship to the number of students, where the number of students is 100 students and the number of academic subjects is 10, and the student can participate in a maximum of three courses, provided that they are with each other. Scheme No. 13 shows the diagram shows how 100 students are distributed among 10 subjects in an alternating manner, explaining the algebraic operations that connect the subjects.

Table (1): shows a comparison between the results of the methods before merging and after merging

comparison between the results before and after the merger					
Metric	Reciprocal ring method	Chart -combinatorial method	model	p-value	f
Quality%	89%	90.10%	%95		
f1-score	85%	88.20%	%90.20		
accuracy	84%	90.30%	95.40%	<0.0001	73.23

It is clear from the figure and outputs that the model's results were the best, with a rate ranging from 5%to 6%, and the model's accuracy was the highest, with a rate ranging from5%11% to, and the model's F1 -score also had the best results compared to the other two methods. At a rate ranging from 2%to5%) Lindgren, B. (2017).

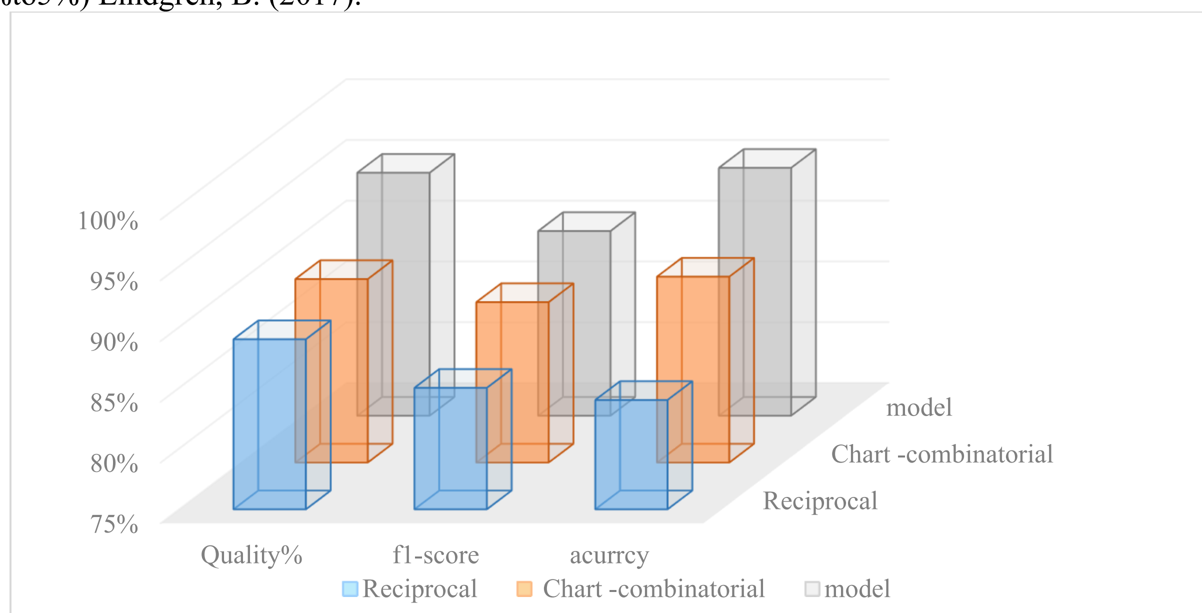

Figure (13): shows a comparison between the results.

It is clear from the figure and the outputs that the model had better results, as the quality of the model was the highest, reaching95% While the quality of the graphical harmonic method was90.1% the quality of the solution of the reciprocating loop method was89%while the accuracy of the model was95.4%and was the highest, as the accuracy of the graphical harmonic method was90%the reciprocating loop method84%as for the index of F1- scores, it reached for the model90.2% As for the solution through the graphical compromise history88.2%and as for the solution through the reciprocal relations method was 85%. (Awange, J. L., & Paláncz, B. (2016).

Table (2): shows a Correlation coefficient between the results of the methods before merging and after merging

	Reciprocal	Chart -combinatorial	model
Reciprocal	1		
Chart -combinatorial	0.244579666	1	
model	0.261235254	0.999851817	1

From the table, the correlation between the graphical method and the harmonic solutions is the most closely related to the model, as the correlation rate reached 0.999.) Zulkoski, E., et.al,2017)

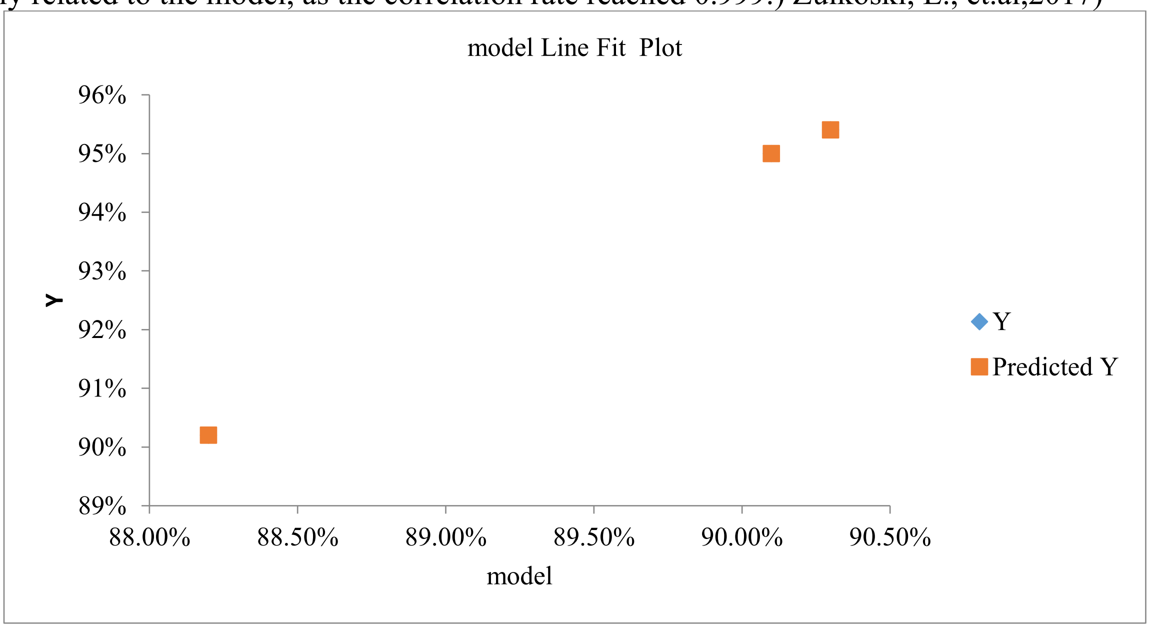

Figure (14): shows model resultsline fit plot.

It is clear from the figure is the fit-line plot curve for the model, and it is clear that the relationship is linear, which means that the data fits the model. It is also noted that there are no significant differences between the expected results and the actual results of the model, which means the success of the model. (Yeats, K. (2017).

5. Conclusions and Recommendation

In this section, the most important conclusions and recommendations that were drawn from the study will be presented Using the theory of commutative rings and graphing harmonic in solutions and approximation of algebraic equations

5.1. Conclusions:

One of the most important conclusions drawn from this study

- Algebraic equations are equations through which the value of the relationship between a group of variables can be found. Using statistical analysis, the direction of this relationship can be determined, whether it is an inverse relationship or a direct relationship. Also, the relationship between the coefficients of the equation can be determined. Using the optimal solution equation, the best solution can be obtained by specifying the coefficients in a more precise way. Accuracy *(Gaborit, P, et.al,2016).*
- The use of machine learning techniques has become very vital and necessary in all applications, as it facilitates obtaining easy and flexible results in record time *(Caminata, A.&Gorla, E,2023)*
- Harmonic methods, graph theory, and commutative ring theory are among the most important mathematical tools that can be used in solving problems of algebraic equations. If these methods are combined in a way that suits the application used, the results obtained will be accurate results, as the advantages of all these methods are obtained at once. These methods can be used after combining them to solve problems with algebraic equations that cannot be solved by traditional algebraic methods. *(Juraev, D. A., & Bozorov, M. N. (2024).*
- By combining combinatorial methods with graph theory and commutative rings, problems of algebraic equations can be solved with greater efficiency and accuracy, ranging from 8 to 20%, than using only combinatorial methods, graphical methods, or commutative rings only.
- Methods of graphing and reciprocal loops Harmonic methods are the most important tools that can be relied upon to obtain accurate results for analysing complex relationships that cannot be solved by traditional methods, especially if they are used in a way that is appropriate for the application in which they are used *(Tanasa, A. (2021).*
- Graph methods, reciprocal loops, and combinatorial solutions are the cornerstone and cornerstone of modern programming and computing sciences, and many programs rely heavily on them *(Knapp, A. W. (2007).*

5.2. Recommendation:

The most important recommendation. that were extracted from this study are a set of important recommendations, which are as follows:

- It is necessary to consider the selection of the optimal methods for integrating graphing methods, harmonic solutions, and reciprocal rings when solving algebraic equations, as well as considering the methods for solving these equations, which are appropriate to the nature of the applications used in them and the nature of the time, requirements, and resources available, whether they are computational resources, human resources, or even technical resources.

- There is a need for more research and studies on integrating methods for solving algebraic equations and creating new hybrid models and methods that can benefit from the potential of each method in terms of time, effort, and cost to solve complex algebraic equations and make things more flexible, especially in applications related to our daily lives. will also make more research and scientific efforts to develop these models and benefit from them to the maximum extent. As integration leads to providing a comprehensive vision of problems, as they can be analysed from more than one perspective, whether it is an algebraic perspective or a geometric perspective, and integration provides powerful and broad tools for solving these problems and can be used in many diverse and broad applications.

Declaration of competing interest

I declare that there is no competing interest with any other authors and that there is no interest or benefit that influenced the results of the research.

References

[1] Rabier, P. J., & Rheinboldt, W. C. (2002). Theoretical and numerical analysis of differential-algebraic equations.

[2] Anderson, D. F., & Lewis, E. F. (2016). A general theory of zero-divisor graphs over a commutative ring. International Electronic Journal of Algebra, 20(20), 111-135.

[3] Spedicato, E. (Ed.). (2012). Computer Algorithms for Solving Linear Algebraic Equations: The State of the Art.

[4] Morey, S., & Villarreal, R. H. (2012). Edge ideals: algebraic and combinatorial properties. Progress in commutative algebra, 1, 85-126.

[5] Yu, D., Yang, B., Wei, Q., Li, A., & Pan, S. (2022). A probabilistic graphical model based on neural-symbolic reasoning for visual relationship detection. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (pp. 10609-10618).

[6] Kamche, H. T., & Kalachi, H. T. (2024). Solving systems of algebraic equations over finite commutative rings and applications. Applicable Algebra in Engineering, Communication and Computing, 1-29.

[7] Epstein, N., Guerrieri, L., & Loper, K. A. (2024). The reciprocal complement of a polynomial ring in several variables over a field. arXiv preprint arXiv:2407.15637.

[8] Diestel, R. (2024). Graph theory. Springer (print edition); Reinhard Diestel (eBooks).

[9] Yan, Z., Dai, H., Wang, Q., & Atluri, S. N. (2023). Harmonic BalanceMethods: A Review and Recent Developments.

[10] Krasnosel'skii, M. A., Vainikko, G. M., Zabreyko, R. P., Ruticki, Y. B., & Stet'senko, V. V. (2012). Approximate solution of operator equations. Springer Science & Business Media.

[11] Kang, H. S. (2010). Analysis of a Reversed Trapezoidal Fin using a 2-D Analytic Method. Journal of the Korean Society for Industrial and Applied Mathematics, 14(3), 151-161.

[12] Pho, K. H. (2022). Improvements of the Newton–Raphson method. Journal of Computational and Applied Mathematics, 408, 114106.
[13] Koochaksaraei, R. H., Meneghini, I. R., Coelho, V. N., & Guimaraes, F. G. (2017). A new visualization method in many-objective optimization with chord diagram and angular mapping. Knowledge-Based Systems, 138, 134-154.
[14] Neukirch, J. (2013). Algebraic number theory (Vol. 322). Springer Science & Business Media.
[15] Judson, T. W. (2020). Abstract algebra: theory and applications.
[16] Allen-Zhu, Z., Li, Y., & Song, Z. (2019, May). A convergence theory for deep learning via over-parameterization. In International conference on machine learning (pp. 242-252). PMLR.
[17] Benedetto, J. J. (2020). Harmonic analysis and applications. CRC Press.
[18] Bain, L. (2017). Statistical analysis of reliability and life-testing models: theory and methods. Routledge.
[19] Tavelli, M., Dumbser, M., Charrier, D. E., Rannabauer, L., Weinzierl, T., & Bader, M. (2019). A simple diffuse interface approach on adaptive Cartesian grids for the linear elastic wave equations with complex topography. Journal of Computational Physics, 386, 158-189.
[20] Lindgren, B. (2017). Statistical theory. Routledge.
[21] Awange, J. L., & Paláncz, B. (2016). Geospatial algebraic computations. Theory and applications, 3rd edn. Springer, Berlin.
[22] Zulkoski, E., Bright, C., Heinle, A., Kotsireas, I., Czarnecki, K., & Ganesh, V. (2017). Combining SAT solvers with computer algebra systems to verify combinatorial conjectures. Journal of Automated Reasoning, 58, 313-339.
[23] Yeats, K. (2017). A combinatorial perspective on quantum field theory. Springer International Publishing.
[24] Gaborit, P., Ruatta, O., Schrek, J.: On the complexity of the rank syndrome decoding problem. IEEE Trans. Inf. Theory 62(2), 1006–1019 (2016)
[25] Caminata, A., Gorla, E.: Solving degree, last fall degree, and related invariants. J. Symb. Comput. 114, 322–335 (2023)
[26] Ambika, S., Mythrae, R., Saranya, S., & Selvanayaki, K. (2019). Algebra in real life. International journal of Trend in scientific research and development (IJTSRD), 3(2), 2456-6470.
[27] Juraev, D. A., & Bozorov, M. N. (2024). The role of algebra and its application in modern sciences. Engineering Applications, 3(1), 59-67.
[28] Tanasa, A. (2021). Combinatorial physics: Combinatorics, quantum field theory, and quantum gravity models. Oxford University Press.
[29] Knapp, A. W. (2007). Advanced algebra. Springer Science & Business Media.