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The Egwaider Type-II Distribution

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Abstract

The aim of this paper is to illustrate the Egwaider type-II (EGW-II) distribution as a new discrete probability distribution that may will have some practical applications in life. The EGW-II distribution is a discrete probability distribution generated as another special case of the finite Egwaider distribution. The properties of this discrete distribution are discussed. Some special cases from the distribution are introduced and some remarks about it are given. Data from this discrete distribution are simulated and used to estimate its shape parameter using two methods of estimation, namely, the maximum likelihood method of estimation and the moments method of estimation. Finally, the conclusion and discussion are given.

Keywords: Power Function Distribution, The Egwaider Distribution, Maximum Likelihood Estimation, Moment Estimation.

1. Introduction

The Egwaider distribution, also named the discrete power function distribution, is a finite three parameter, univariate, unimodal discrete probability distribution obtained by Muiftah [Ref. 2] as a discrete analogue of the continuous power function distribution using a well known method of discretization and is given by the probability mass function (pmf):

$$P(Y=y)=\begin{cases}\left(\frac{y-a}{s}\right)^{\theta}-\left(\frac{y-a+1}{s}\right)^{\theta}, & \frac{s}{\theta}<0\\ \left(\frac{y-a+1}{s}\right)^{\theta}-\left(\frac{y-a}{s}\right)^{\theta}, & \frac{s}{\theta}>0\end{cases};$$

with a finite or semi-infinite support defined according to the signs of both β and s as follows:

$$\begin{aligned}&y\in[a,a+s-1], \quad s>0,\theta>0;\\&y\in[a+s,\,a-1], \quad s<0,\theta>0;\\&y\in[a+s,\infty), \quad s>0,\theta<0;\\&y\in(-\infty,a+s], \quad s<0,\theta<0.\end{aligned}$$

where, a and $a+s$ are the end points of the support of the distribution, and β is the shape parameter. Thus, the Egwaider distribution can be re-written as:

$$P(Y=y)=\begin{cases}\frac{(y-a)^{\theta}-(y-a+1)^{\theta}}{s^{\theta}}, & \frac{s}{\theta}<0\\ \frac{(y-a+1)^{\theta}-(y-a)^{\theta}}{s^{\theta}}, & \frac{s}{\theta}>0\end{cases};$$

$$\begin{aligned}&y=a,a+1,\cdots,a+s-1; && s>0,\theta>0\\&y=a+s,a+s+1,\cdots,a-1; && s<0,\theta>0\\&y=a+s,a+s+1,\cdots; && s>0,\theta<0\\&y=\cdots,a+s-1,a+s; && s<0,\theta<0\end{aligned} \qquad (1)$$

The Egwaider type-I (EGW-I) distribution was obtained by Muiftah [Ref. 3], by considering θ > 0 and s > 0 in the pmf (1), thus its pmf is given by:

$$P(T=t)=\frac{(t-a+1)^{\theta}-(t-a)^{\theta}}{s^{\beta}}, t=a,a+1,\cdots,a+s-1; \quad s>0;\theta>0.$$

2. Derivation of the Egwaider Type-II Distribution

Taking $\theta > 0$ and $s < 0$ in the pmf (1), the Egwaider type-II (EGW-II) distribution arises and is given by the following pmf:

$$P(X=x)=\frac{(x-a)^{\theta}-(x-a+1)^{\theta}}{s^{\theta}}, \quad x=a+s,a+s+1,\cdots,a-1; \quad s<0; \quad \theta>0 \tag{2}$$

where, both a and s are integers.

It may be proved that the function (2) is a pmf as follows:

$$\sum_{x=a+s}^{a-1}\frac{(x-a)^{\theta}-(x-a+1)^{\theta}}{s^{\theta}}=\frac{1}{s^{\theta}}\{[(s)^{\theta}-(s+1)^{\theta}]+[(s+1)^{\theta}-(s+2)^{\theta}]+\cdots+[(-2)^{\theta}-(-1)^{\theta}]+[(-1)^{\theta}-(0)^{\theta}]\}$$

$$=\frac{1}{s^{\theta}}[(s)^{\theta}-(0)^{\theta}]=\frac{s^{\theta}}{s^{\theta}}=1.$$

2.1. Cumulative Distribution Function:

The cumulative distribution function (cdf) of the EGW-II distribution is given by:

$$F(x)=1-\frac{(x-a+1)^{\theta}}{s^{\theta}}, \quad x=a+s,\cdots,a-1; \quad s<0 \tag{3}$$

Proof:

$$F(x)=\sum_{u=a+s}^{x}\frac{(u-a)^{\theta}-(u-a+1)^{\theta}}{s^{\theta}}$$

$$=\frac{[(s)^{\theta}-(s+1)^{\theta}]+[(s+1)^{\theta}-(s+2)^{\theta}]+\cdots+[(x-a-1)^{\theta}-(x-a)^{\theta}]+[(x-a)^{\theta}-(x-a+1)^{\theta}]}{s^{\theta}}$$

$$=\frac{s^{\theta}-(x-a+1)^{\theta}}{s^{\theta}}=1-\frac{(x-a+1)^{\theta}}{s^{\theta}}. \qquad \#$$

2.2. Survival Function:

The survival function (sf) of the EGW-II distribution is hence given by:

$$S(x)=1-F(x)=\frac{(x-a+1)^{\theta}}{s^{\theta}}, \quad x=a+s,\cdots,a-1; \quad s<0, \tag{4}$$

It may be observed that the EGW-II distribution reserve the same (cdf / sf) of the continuous power function distribution when $\theta > 0$ and s< 0 [Ref. 1,2].

2.3. Failure Rate Function:

The failure rate function (frf) of the EGW-II distribution is given by:

$$h(\mathrm{x}) = \frac{(x-a)^{\theta} - (x-a+1)^{\theta}}{(x-a)^{\theta}}, \quad x = a+s, \cdots, a-1; \quad s<0, \qquad \textbf{(5)}$$

Proof:

$$h(\mathrm{x}) = \frac{S(y-1)-S(y)}{S(y-1)} = \frac{[\frac{(x-a)^{\theta}}{s^{\theta}} - \frac{(x-a+1)^{\theta}}{s^{\theta}}]}{[\frac{(x-a)^{\theta}}{s^{\theta}}]} = \frac{[\frac{(x-a)^{\theta}-(x-a+1)^{\theta}}{s^{\theta}}]}{[\frac{(x-a)^{\theta}}{s^{\theta}}]} = \frac{(x-a)^{\theta}-(x-a+1)^{\theta}}{(x-a)^{\theta}}. \qquad \#$$

2.4. Moments:

The rth moment of the EGW-II distribution is given by:

$$\mu_r' = E(X^r) = \frac{1}{s^{\theta}} \sum_{i=s}^{-1} (a+i)^r [(i)^{\theta} - (i+1)^{\theta}]; \quad s<0, \qquad \textbf{(6)}$$

Proof:

$$\mu_r' = E(X^r) = \frac{1}{s^{\theta}} \sum_{y=a+s}^{a-1} x^r [(x-a)^{\theta} - (x-a+1)^{\theta}]$$

$$= \frac{1}{s^{\theta}} \{(a+s)^r [(s)^{\theta} - (s+1)^{\theta}] + (a+s+1)^r [(s+1)^{\theta} - (s+2)^{\theta}] + \cdots + (a-1)^r [(-1)^{\theta} - (0)^{\theta}]\}$$

$$= \frac{1}{s^{\theta}} \sum_{i=s}^{-1} (a+i)^r [(i)^{\theta} - (i+1)^{\theta}] \qquad \#$$

It may observed that $\mu_0' = E(X^0) = 1$, as $\sum_{i=s}^{-1} [(i)^{\theta} - (i+1)^{\theta}] = s^{\theta}$,

Proof:

$$\sum_{i=s}^{-1} [(i)^{\theta} - (i+1)^{\theta}] = [(s)^{\theta} - (s+1)^{\theta}] + [(s+1)^{\theta} - (s+2)^{\theta}] + \cdots + [(-2)^{\theta} - (-1)^{\theta}] + [(-1)^{\theta} - (0)^{\theta}]$$

$$= s^{\theta} - (0)^{\theta} = s^{\theta} \qquad \#$$

2.4.1. Mean:

The mean of the EGW-II distribution is given by:

$$\mu_1' = \mu_x = E(X) = a + \frac{\sum_{i=s}^{-1} [(i)^{\theta+1} - i(i+1)^{\theta}]}{s^{\theta}}; \quad s < 0 \tag{7}$$

Proof:

$$\mu_1' = E(X) = \frac{1}{s^{\theta}} \sum_{i=s}^{-1} (a+i)[(i)^{\theta} - (i+1)^{\theta}]$$

$$= \frac{1}{s^{\theta}} \sum_{i=s}^{-1} [a(i)^{\theta} - a(i+1)^{\theta} + (i)^{\theta+1} - (i)(i+1)^{\theta}]$$

$$= \frac{1}{s^{\theta}} \{[a(s)^{\theta} - a(s+1)^{\theta} + (s)^{\theta+1} - (s)(s+1)^{\theta}] + [a(s+1)^{\theta} - a(s+2)^{\theta} + (s+1)^{\theta+1} - (s+1)(s+2)^{\theta}]$$
$$+ [a(s+2)^{\theta} - a(s+3)^{\theta} + (s+2)^{\theta+1} - (s+2)(s+3)^{\theta}] + \cdots + [a(-2)^{\theta} - a(-1)^{\theta} + (-2)^{\theta+1} - (-2)(-1)^{\theta}]$$
$$+ [a(-1)^{\theta} - a(0)^{\theta} + (-1)^{\theta+1} - (-1)(0)^{\theta}]\}$$

$$= \frac{a(s)^{\theta} + \{[(s)^{\theta+1} - (s)(s+1)^{\theta}] + [(s+1)^{\theta+1} - (s+1)(s+2)^{\theta}] + [(s+2)^{\theta+1} - (s+2)(s+3)^{\theta}] + \cdots + [(-2)^{\theta+1} - (-2)(-1)^{\theta}] + [(-1)^{\theta+1} - (-1)(0)^{\theta}]\}}{s^{\theta}}$$

$$= \frac{a(s)^{\theta} + \sum_{i=s}^{-1} [(i)^{\theta+1} - i(i+1)^{\theta}]}{s^{\theta}} = a + \frac{\sum_{i=s}^{-1} [(i)^{\theta+1} - i(i+1)^{\theta}]}{s^{\theta}} \qquad \#$$

2.4.2. Second Moment:

The second moment of the EGW-II distribution is given by:

$$\mu_2' = E(X^2) = a^2 + \frac{\sum_{i=s}^{-1} \{(2a+i)[(i)^{\theta+1} - i(i+1)^{\theta}]\}}{s^{\theta}}; \quad s < 0 \tag{8}$$

Proof:

$$\mu_2' = E(X^2) = \frac{1}{s^\theta}\sum_{i=s}^{-1}(a+i)^2[(i)^\theta-(i+1)^\theta] = \frac{1}{s^\theta}\sum_{i=s}^{-1}(a^2+2ai+i^2)[(i)^\theta-(i+1)^\theta]$$

$$= \frac{1}{s^\theta}\sum_{i=s}^{-1}[a^2(i)^\theta - a^2(i+1)^\theta + 2a(i)^{\theta+1} - 2ai(i+1)^\theta + (i)^{\theta+2} - i^2(i+1)^\theta]$$

$$= \frac{1}{s^\theta}\{[a^2(s)^\theta - a^2(s+1)^\theta + 2a(s)^{\theta+1} - 2a(s)(s+1)^\theta + (s)^{\theta+2} - (s)^2(s+1)^\theta]$$
$$+[a^2(s+1)^\theta - a^2(s+2)^\theta + 2a(s+1)^{\theta+1} - 2a(s+1)(s+2)^\theta + (s+1)^{\theta+2} - (s+1)^2(s+2)^\theta] + \cdots$$
$$\cdots + [a^2(-2)^\theta - a^2(-1)^\theta + 2a(-2)^{\theta+1} - 2a(-2)(-1)^\theta + (-2)^{\theta+2} - (-2)^2(-1)^\theta]$$
$$+[a^2(-1)^\theta - a^2(0)^\theta + 2a(-1)^{\theta+1} - 2a(-1)(0)^\theta + (-1)^{\theta+2} - (-1)^2(0)^\theta]\}$$

$$= \frac{a^2(s)^\theta}{s^\theta} + \frac{\begin{array}{l}\{[2a(s)^{\theta+1} - 2a(s)(s+1)^\theta + (s)^{\theta+2} - (s)^2(s+1)^\theta] \\ +[2a(s+1)^{\theta+1} - 2a(s+1)(s+2)^\theta + (s+1)^{\theta+2} - (s+1)^2(s+2)^\theta] + \cdots \\ \cdots + [2a(-2)^{\theta+1} - 2a(-2)(-1)^\theta + (-2)^{\theta+2} - (-2)^2(-1)^\theta] + [2a(-1)^{\theta+1} + (-1)^{\theta+2}]\}\end{array}}{s^\theta}$$

$$= a^2 + \frac{\sum_{i=s}^{-1}\{2a(i)^{\theta+1} - 2ai(i+1)^\theta + (i)^{\theta+2} - i^2(i+1)^\theta\}}{s^\theta}$$

$$= a^2 + \frac{\sum_{i=s}^{-1}\{(2a+i)[(i)^{\theta+1} - i(i+1)^\theta]\}}{s^\theta} \qquad \#$$

2.5. Variance:

Using the first and second moments given by (7) and (8), the variance of the EGW-II distribution will be:

$$\mathrm{Var}(X) = \frac{\sum_{i=s}^{-1}[(i)^{\theta+2} - i^2(i+1)^\theta]}{s^\theta} - \frac{\left[\sum_{i=s}^{-1}[(i)^{\theta+1} - i(i+1)^\theta]\right]^2}{s^{2\theta}}; s<0 \qquad \textbf{(9)}$$

Proof:

$$\mathrm{Var}(X)=E(X^2)-[E(X)]^2 = a^2+\frac{\sum_{i=s}^{-1}\{(2a+i)[(i)^{\theta+1}-i(i+1)^{\theta}]\}}{s^{\theta}}-\left[a+\frac{\sum_{i=s}^{-1}[(i)^{\theta+1}-i(i+1)^{\theta}]}{s^{\theta}}\right]^2$$

Let $c_i=(i)^{\theta+1}-i(i+1)^{\theta},$ **(10)**

then, $$\mathrm{Var}(X)=a^2+\frac{\sum_{i=s}^{-1}(2a+i)c_i}{s^{\theta}}-a^2-\frac{2a\sum_{i=s}^{-1}c_i}{s^{\theta}}-\left(\frac{\sum_{i=s}^{-1}c_i}{s^{\theta}}\right)^2=\frac{2a\sum_{i=s}^{-1}c_i}{s^{\theta}}+\frac{\sum_{i=s}^{-1}ic_i}{s^{\theta}}-\frac{2a\sum_{i=s}^{-1}c_i}{s^{\theta}}-\frac{\left(\sum_{i=s}^{-1}c_i\right)^2}{s^{2\theta}}$$

$$\therefore \quad \mathrm{Var}(X)=\frac{\sum_{i=s}^{-1}ic_i}{s^{\theta}}-\frac{\left(\sum_{i=s}^{-1}c_i\right)^2}{s^{2\theta}}$$ **(11)**

replacing c_i by its value given by (10) in (11), $$\mathrm{Var}(X)=\frac{\sum_{i=s}^{-1}[(i)^{\theta+2}-i^2(i+1)^{\theta}]}{s^{\theta}}-\frac{\left[\sum_{i=s}^{-1}[(i)^{\theta+1}-i(i+1)^{\theta}]\right]^2}{s^{2\theta}}$$ #

2.6. Mode:

The mode of the EGW-II distribution is analogues to that of the continuous power function distribution when s < 0 and is given as:

$$\mathrm{Mode}\ (X)=\begin{cases} a-1, & \theta<1 \\ a+s, & \theta>1 \end{cases} \quad ;$$ **(12)**

Proof:

As $s < 0$, the pmf (6) attains its maximum value if the nominator $[(x-a)^{\theta}-(x-a+1)^{\theta}]$ is maximum for $a+s \le x \le a-1$.

For $\theta < 1$: $[(x-a)^{\theta} - (x-a+1)^{\theta}]$ is maximum if X takes its minimum value ($x = a - 1$),

$$\Rightarrow \underset{\theta<1}{Max}\, P(X=x) = P(X=a-1) = \frac{(a-1-a)^{\theta} - (a-1-a+1)^{\theta}}{s^{\theta}} = \frac{(-1)^{\theta} - (0)^{\theta}}{s^{\theta}} = \frac{(-1)^{\theta}}{s^{\theta}}, \qquad s<0$$

For $\theta > 1$: $[(x-a)^{\theta} - (x-a+1)^{\theta}]$ is maximum if X takes its maximum value ($x = a+s$),

$$\Rightarrow \underset{\theta>1}{Max}\, P(X=x) = P(X=a+s) = \frac{(a+s-a)^{\theta} - (a+s-a+1)^{\theta}}{s^{\theta}} = \frac{(s)^{\theta} - (s+1)^{\theta}}{s^{\theta}}, \qquad s<0$$

3. Some Special Cases of the Egwaider Type-II Distribution

Analogues to the special cases from the continuous power function distribution proposed by Crooks [Ref.1], some discrete distributions may be considered as special cases from the EGW-II distribution as follows:

3.1. For $\theta = 1$, the EGW-II distribution given by the pmf (2) reduces to the discrete uniform distribution in the interval [$a+s$, a-1], with the pmf:

$$P(X=x) = \frac{(x-a)-(x-a+1)}{s} = \frac{1}{s}, \qquad x = a+s, \cdots, a-1; \quad s<0, \tag{13}$$

3.2. For $\theta = 2$, the EGW-II distribution given by the pmf (2) reduces to the discrete descending wedge (left rectangular) distribution in the interval [$a+s$, a-1], with a pmf:

$$P(X=x) = \frac{(x-a)^2 - (x-a+1)^2}{s^2} = \frac{(x^2-2ax+a^2)-(x^2+a^2-2ax+2x-2a+1)}{s^2}$$
$$= \frac{2a-2x+1}{s^2}, \qquad x = a+s, \cdots, a-1; \quad s<0 \tag{14}$$

3.3. For $a = 0$, $0 < \theta < 1$, the EGW-I distribution given by the pmf (2) reduces to the discrete analogue of the second type of the Pearson type VIII distribution with a pmf:

$$P(X=x) = \frac{(x-0)^{\theta} - (x-0+1)^{\theta}}{s^{\theta}} = \frac{(x)^{\theta} - (x+1)^{\theta}}{s^{\theta}}, \quad x = s, \cdots, -1; \qquad s<1, \quad 0<\theta<1 \tag{15}$$

3.4. For $a = 0$, $\theta > 1$, the EGW-II distribution given by the pmf (2) reduces to the discrete analogue of the second type of the Pearson type IX distribution with a pmf:

$$P(X = x) = \frac{(x)^{\theta} - (x+1)^{\theta}}{s^{\theta}}, \qquad x = s, \cdots, -1; \qquad s < 1, \quad \theta > 1 \qquad (16)$$

4. Practical Applications

4.1. Table (1) represents the pmf [$P(x)$], the cdf [$F(x)$], the sf [$S(x)$], and the frf [$h(x)$] of the EGW-II (0, -10, 0.2) distribution, whereas, figures (1) and (2) represent respectively the pmf [$P(x)$] and the frf [$h(x)$] of the EGW-II (0, -10, 0.2) distribution.

$$P(X = x) = \frac{x^{0.2} - (x+1)^{0.2}}{(-10)^{0.2}}, \quad x = -10, -9, \cdots, -1$$

This is the discrete analogue of the second type of the Pearson type VIII distribution given by (15) above.

Table (1): The EGW-II (0, -10, 0.2) distribution

x	$P(x)$	$F(x)$	$S(x)$	$h(x)$
-10	0.020852	0.020852	0.979148	0.020852
-9	0.022796	0.043648	0.956352	0.023281
-8	0.025203	0.068850	0.931150	0.026353
-7	0.028269	0.097120	0.902880	0.030360
-6	0.032330	0.129449	0.870551	0.035807
-5	0.037997	0.167447	0.832553	0.043648
-4	0.046550	0.213997	0.786003	0.055912
-3	0.061223	0.275220	0.724780	0.077892
-2	0.093822	0.369043	0.630957	0.129449
-1	0.630957	1	0	1
Σ	1.000000			

E(X) = -2.3856249 , Var(X) = 5.65446081

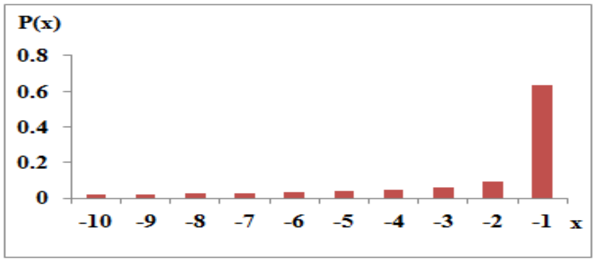

Fig (1): The pmf of the EGW-II (0, -10. 0.2) distribution

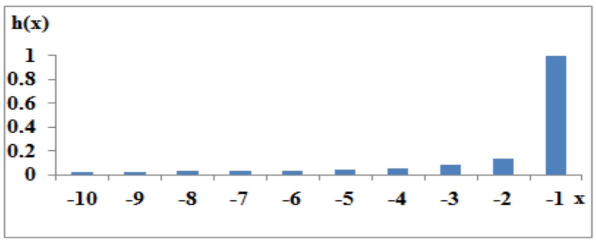

Fig (2): The frf of the EGW-II (0, -10. 0.2) distribution

4.2. Table (2) represents the pmf, the cdf, the sf, and the frf, of the EGW-II (5, -8, 0.2) distribution, whereas, figures (3) and (4) represent respectively the pmf and the frf of the EGW-II (5, -8, 0.2) distribution.

$$P(X=x)=\frac{(x-5)^{0.2}-(x-4)^{0.2}}{(-8)^{0.2}}, \quad x=-3,-2,\cdots,4$$

Table (2): The EGW-II (5, -8, 0.2) distribution

x	$P(x)$	$F(x)$	$S(x)$	$h(x)$
-3	0.026353	0.026353	0.973647	0.026353
-2	0.029560	0.055912	0.944088	0.030360
-1	0.033805	0.089718	0.910282	0.035807
0	0.039732	0.129449	0.870551	0.043647
1	0.048675	0.178124	0.821876	0.055912
2	0.064018	0.242142	0.757858	0.077892
3	0.098104	0.340246	0.659754	0.129449
4	0.659754	1	0	1
Σ	1.000000			

E(X) = 2.93805551, Var(X) = 3.500796693

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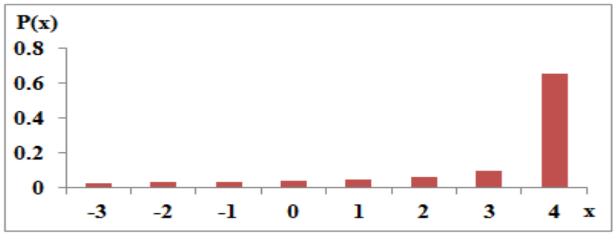

Fig (3): The pmf of the EGW-II (5, -8. 0.2) distribution

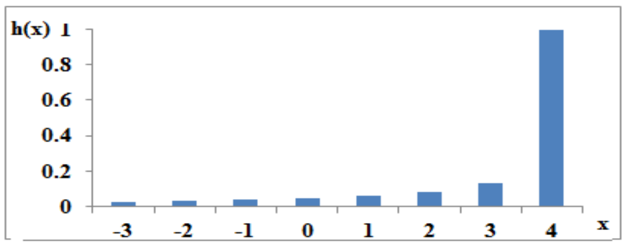

Fig (4): The frf of the EGW-II (5, -8. 0.2) distribution

4.3. Table (3) represents the pmf, the cdf, the sf, and the frf, of the EGW-II (0, -10, 1.2) distribution, whereas, figures (5) and (6) represent respectively the pmf and the frf of the EGW-II (0, -10, 1.2) distribution.

$$P(X = x) = \frac{x^{1.2} - (x-1)^{1.2}}{(-10)^{1.2}}, \quad x = -10, -9, \cdots, -1$$

This is the discrete analogue of the second type of the Pearson type IX distribution given by (16) above.

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Table (3): The EGW-II (0, -10, 1.2) distribution

x	$P(x)$	$F(x)$	$S(x)$	$h(x)$
-10	0.118766	0.118766	0.881234	0.118766
-9	0.116152	0.234918	0.765082	0.131806
-8	0.113277	0.348195	0.651805	0.148059
-7	0.110077	0.458272	0.541728	0.168880
-6	0.106453	0.564725	0.435275	0.196506
-5	0.102254	0.666979	0.333021	0.234918
-4	0.097220	0.764199	0.235801	0.291934
-3	0.090845	0.855044	0.144956	0.385261
-2	0.081860	0.936904	0.063096	0.564725
-1	0.063096	1	0	1
Σ	1.000000			

E(X) = -5.9480021, Var(X) = 7.70200534

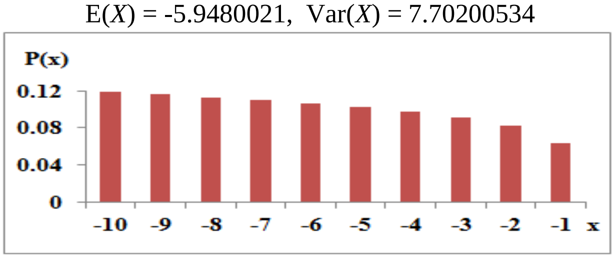

Fig (5): The pmf of the EGW-II (0, -10. 1.2) distribution

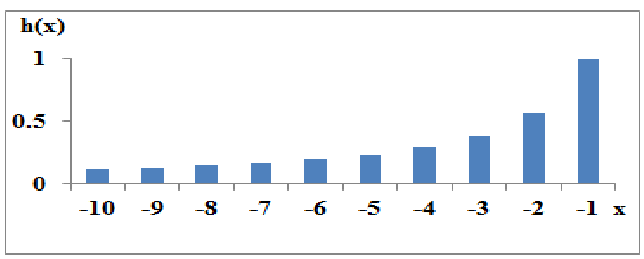

Fig (6): The frf of the EGW-II (0, -10. 1.2) distribution

4.4. Table (4) represents the pmf, the cdf, the sf, and the frf, of the EGW-II (3, -9, 3.4) distribution, whereas, figures (7) and (8) represent respectively the pmf and the frf of the EGW-II (3, -9, 3.4) distribution.

$$P(X=x)=\frac{(x-3)^{3.4}-(x-2)^{3.4}}{(-9)^{3.4}}, \qquad x=-6,-5,\cdots,2$$

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Table (4): The EGW-II (3, -9, 3.4) distribution

x	$P(x)$	$F(x)$	$S(x)$	$h(x)$
-6	0.329990	0.329990	0.670010	0.329990
-5	0.244501	0.574491	0.425509	0.364922
-4	0.173573	0.748064	0.251936	0.407919
-3	0.116394	0.864458	0.135542	0.461998
-2	0.072070	0.936528	0.063472	0.531720
-1	0.039605	0.976134	0.023866	0.623982
0	0.017854	0.993987	0.006013	0.748064
1	0.005443	0.999430	0.000570	0.905268
2	0.000570	1	0	1
Σ	1.000000			

E(X) = -4.423082696, Var(X) = 2.588826734

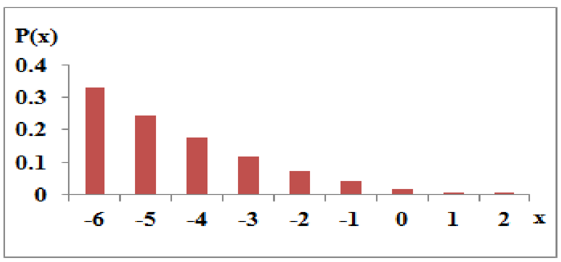

Fig (7): The pmf of the EGW-I (3, -9. 3.4) distribution

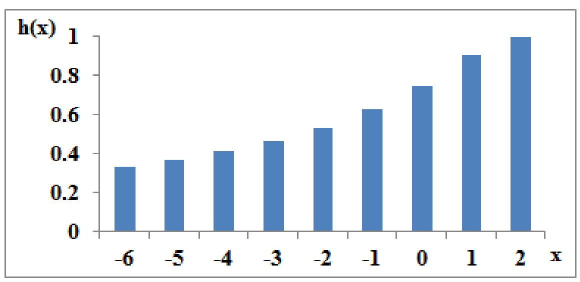

Fig (8): The frf of the EGW-I (3, -9. 3.4) distribution

4.5. Table (5) represents the pmf, the cdf, the sf, and the frf, of the EGW-II (-5, -8, 2.6) distribution, whereas, figures (9) and (10) represent respectively the pmf and the frf of the EGW-II (-5, -8, 2.6) distribution.

$$P(X=x)=\frac{(x+5)^{2.6}-(x+6)^{2.6}}{(-8)^{2.6}}, \quad x=-13,-12,\cdots,-6$$

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Table (5): The EGW-II (-5, -8, 2.6) distribution

x	$P(x)$	$F(x)$	$S(x)$	$h(x)$
-13	0.293323	0.293323	0.706677	0.293323
-12	0.233352	0.526675	0.473325	0.33021
-11	0.178687	0.705362	0.294638	0.377515
-10	0.129699	0.835062	0.164938	0.440199
-9	0.086869	0.921931	0.078069	0.526675
-8	0.050865	0.972795	0.027205	0.651532
-7	0.022718	0.995513	0.004487	0.835062
-6	0.004487	1	0	1
Σ	1.000000			

E(X) = -11.25066091 , Var(X) = 2.75443063

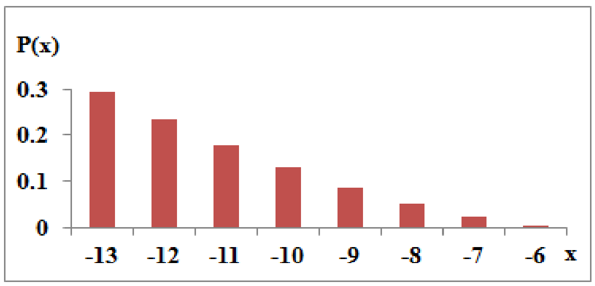

Fig (9): The pmf of the EGW-II (-5, -8. 2.6) distribution

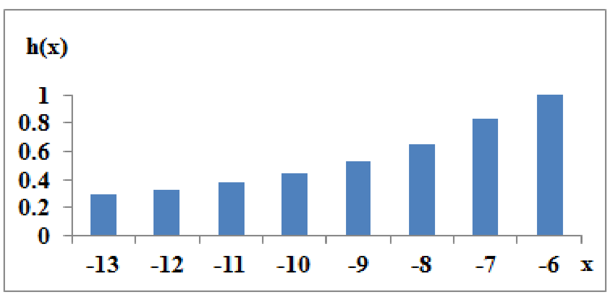

Fig (10): The frf of the EGW-II (-5, -8. 2.6) distribution

4.6. Table (6) represents the pmf, the cdf, the sf, and the frf, of the EGW-II (8, -2, 0.4) distribution, whereas, figures (11) and (12) represent respectively the pmf and the frf of the EGW-II (8, -2, 0.4) distribution.

$$P(X=x)=\frac{(x-8)^{0.4}-(x-7)^{0.4}}{(-8)^{0.4}}, \quad x=6,7.$$

Table (6): The EGW-II (8, -2, 0.4) distribution

x	$P(x)$	$F(x)$	$S(x)$	$h(x)$
6	0.24214172	0.24214172	0.75785828	0.24214172
7	0.75785828	1	0	1
Σ	1.000000			

E(X) = 6.75785828, Var(X) = 0.18350910582

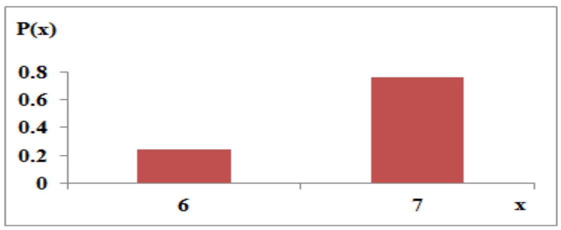

Fig (11): The pmf of the EGW-II (8, -2, 0.4) distribution

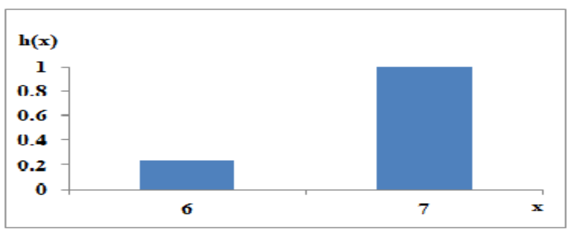

Fig (12): The frf of the EGW-II (8, -2, 0.4) distribution

4.7. Table (7) represents the pmf, the cdf, the sf, and the frf, of the EGW-II (-8, -5, 0.2) distribution, whereas, figures (13) and (14) represent respectively the pmf and the frf of the EGW-II (-8, -5, 0.2) distribution.

$$P(X=x)=\frac{(x+8)^{0.2}-(x+9)^{0.2}}{(-5)^{0.2}}, \quad x=-13,-12,\cdots,-9$$

Table (7): The EGW-II (-8, -5, 0.2) distribution

x	$P(x)$	$F(x)$	$S(x)$	$h(x)$
-13	0.0436475	0.0436475	0.9563525	0.0436475
-12	0.05347205	0.09711955	0.90288045	0.05591249
-11	0.07032724	0.16744679	0.83255321	0.07789209
-10	0.10777354	0.27522034	0.72477966	0.12944944
-9	0.72477966	1	0	1
Σ	1.00000000			

$$E(X) = -9.5834342, \quad Var(X) = 1.228295519$$

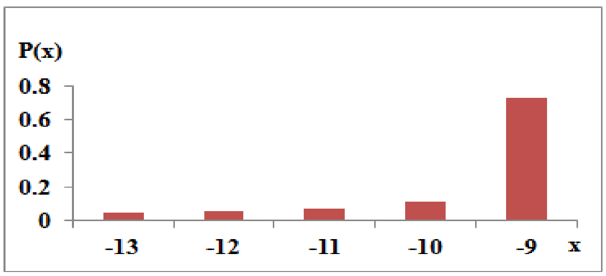

Fig (13): The pmf of the EGW-II (-8, -5, 0.2) distribution

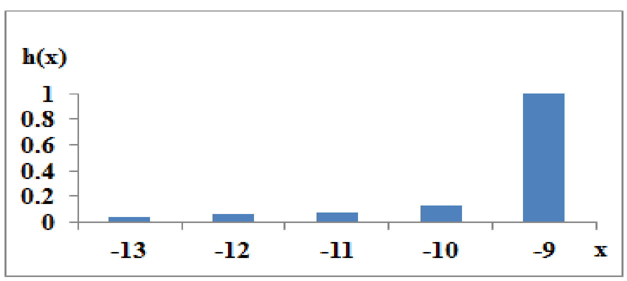

Fig (14): The frf of the EGW-II (-8, -5, 0.2) distribution

4.8. Table (8) represents the pmf, the cdf, the sf, and the frf, of the EGW-II (5, -8, 0.3) distribution, whereas, figures (15) and (16) represent respectively the pmf and the frf of the EGW-II (5, -8, 0.3) distribution.

$$P(X=x) = \frac{(x-5)^{0.3} - (x-4)^{0.3}}{(-8)^{0.3}}, \quad x = -3, -2, \cdots, 4$$

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Table (8): The EGW-II (5, -8, 0.3) distribution

x	$P(x)$	$F(x)$	$S(x)$	$h(x)$
-3	0.03926765	0.03926765	0.96073235	0.03926765
-2	0.04341760	0.08268525	0.91731475	0.04519219
-1	0.04882639	0.13151163	0.86848837	0.05322752
0	0.05623597	0.18774760	0.81225240	0.06475155
1	0.06716129	0.25490889	0.74509111	0.08268525
2	0.08533715	0.34024604	0.65975396	0.11453251
3	0.12386722	0.46411327	0.53588673	0.1877476
4	0.53588673	1	0	1
Σ	1.00000000			

$E(X) = 2.499519664$, $Var(X) = 4.425809669$

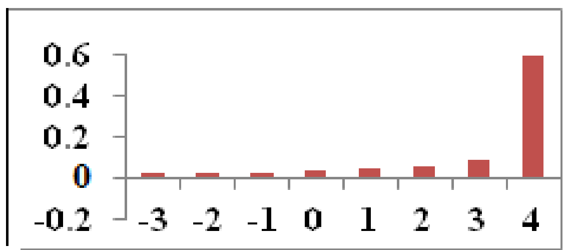

Fig (15): The pmf of the EGW-II (5, -8, 0.3) distribution

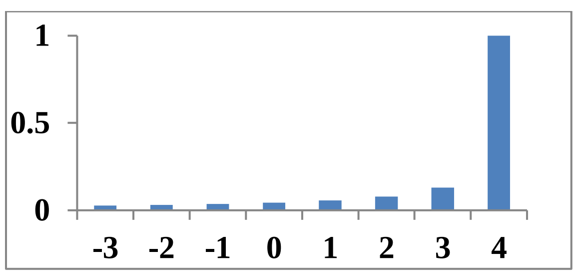

Fig (16): The frf of the EGW-II (5, -8, 0.3) distribution

The plots of the pmf 's given by figures (1), (3), (5), (7), (9), (11), (13) and (15) show either increasing or decreasing shapes according to the values of a, s, and θ. It may observed that the mode of the EGW-II distribution, may be given as in (12).

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The plots given by figures (2), (4), (6), (8), (10), (12), (14) and (16), show the different shapes of the frf of the EGW-I distribution according to the values of *a*, *s*, and θ, some shapes of the frf may be observed including bathtub and j shape frf.

5. Estimation of the Parameter of the Distribution

The values of a and s are usually pre-assigned (can be obtained from the data), therefore, θ (the shape parameter of the EGW-II distribution) is the only parameter to be estimated.

5.1. Maximum likelihood estimation:

Given a random sample $X_1, X_2, ..., X_n$ from the EGW-II distribution (2), then the likelihood function of this sample is:

$$\mathrm{L}(a,s,\theta)=\prod_{i=1}^{n}\left[\frac{(x_i-a)^{\theta}-(x_i-a+1)^{\theta}}{s^{\theta}}\right],\quad s<0, \tag{17}$$

The log - likelihood function is then given by:

$$\ln \mathrm{L}(a,s,\theta)=\sum_{i=1}^{n}\{\ln[(x_i-a)^{\theta}-(x_i-a+1)^{\theta}]\}-n\theta\ln(s),\quad s<0, \tag{18}$$

Maximizing (18), to get the MLE $\hat{\theta}$ of the parameter θ, needs numerical technique.

5.2. Moment estimation:

Given a random sample $X_1, X_2, ..., X_n$ from the EGW-II distribution (2), then, the moment estimate $\tilde{\theta}$ of the parameter θ may be estimated using the method of moments by solving the equation $\mu_1'=m_1$, where, $\mu_1'=E(X)$, and m_1 is the first moment of the sample (sample mean) given by: $m_1=\frac{1}{n}\sum_{j=1}^{n}x_j$, then, for the EGW-II distribution:

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$$a+\frac{\sum_{j=s}^{-1}[(j)^{\tilde{\theta}+1}-j(j+1)^{\tilde{\theta}}]}{s^{\tilde{\theta}}}=\frac{1}{n}\sum_{j=1}^{n}x_j \quad (19)$$

Equation (19) also need a numerical technique to be solved to get the moment estimate $\tilde{\theta}$ of the parameter θ.

5.3. Simulation study:

A Mathcad program is used to simulate data from the EGW-II distribution for some arbitrary proposed values of a, s, and θ. The ML and moment estimates of θ ($\hat{\theta}$ and $\tilde{\theta}$) and their corresponding variances and MSEs are obtained for sample sizes (n = 25, 50, and 100) using 1000 replications. The obtained results are given in table (9).

6. Discussion and Conclusion

The EGW-II distribution is illustrated as a new discrete probability distribution. The properties of the distribution were discussed. Some practical applications (examples) considering different values of the parameters of the distribution were given. The ML and moment estimates of the shape parameter θ and their corresponding variances and MSEs are obtained for different sample sizes using simulated data from EGW-II distribution.

An inspection on table (9) may show that the ml method yielded an over estimation whereas the method of moments yielded an under estimation for the shape parameter θ. The variances and the MSEs of the ML estimates seem to be smaller in most cases than those of the moment estimates, which indicates that the ML estimates seem to be better to use than the moment estimates. It may be also observed that the estimates in both cases become closer to the true parameter value by increasing the sample size, which indicates that both estimators are consistent.

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Table(9): ML and moment estimation of the parameter θ of the EGW-II (*a*, *s*, *θ*) distribution

***a* = 0 , *s* = -10**

θ	n	ML estimation			Moment estimation		
		$\hat{\theta}$	Var $(\hat{\theta})$	MSE $(\hat{\theta})$	$\tilde{\theta}$	Var $(\tilde{\theta})$	MSE $(\tilde{\theta})$
0.2	25	**0.308**	0.024	0.025	**0.116**	0.005	0.020
	50	**0.276**	0.020	0.020	**0.153**	0.002	0.018
	100	**0.219**	0.013	0.014	**0.186**	0.001	0.016
0.5	25	**0.617**	0.005	0.306	**0.431**	0.015	0.015
	50	**0.594**	0.003	0.314	**0.456**	0.007	0.008
	100	**0.533**	0.001	0.311	**0.483**	0.004	0.004
0.8	25	**0.909**	0.014	0.079	**0.744**	0.027	0.031
	50	**0.875**	0.008	0.069	**0.751**	0.013	0.021
	100	**0.829**	0.004	0.064	**0.768**	0.006	0.014
1.0	25	**1.107**	0.023	0.046	**0.902**	0.038	0.057
	50	**1.054**	0.013	0.044	**0.945**	0.019	0.041
	100	**1.019**	0.006	0.045	**0.979**	0.009	0.032
1.3	25	**1.406**	0.009	0.094	**1.159**	0.050	0.090
	50	**1.380**	0.002	0.099	**1.205**	0.024	0.066
	100	**1.324**	0.0005	0.101	**1.264**	0.011	0.060
1.6	25	**1.713**	0.003	0.093	**1.391**	0.072	0.159
	50	**1.682**	0.001	0.091	**1.475**	0.035	0.136
	100	**1.602**	0.001	0.091	**1.568**	0.018	0.126
2.0	25	**2.158**	0.041	0.080	**1.862**	0.117	0.346
	50	**2.088**	0.026	0.067	**1.914**	0.054	0.302
	100	**2.016**	0.017	0.061	**1.941**	0.028	0.286
2.4	25	**2.521**	0.061	0.125	**2.229**	0.162	0.605
	50	**2.486**	0.032	0.090	**2.286**	0.073	0.522
	100	**2.409**	0.022	0.088	**2.323**	0.036	0.530
3.0	25	**3.179**	0.076	0.174	**2.770**	0.221	1.015
	50	**3.096**	0.042	0.121	**2.877**	0.109	0.884
	100	**3.012**	0.021	0.096	**2.955**	0.048	0.881

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Table(9) continued: ML and moment estimation of the parameter β of the EGW-I (*a, s, θ*) distribution

***a* = 4 , *s* = -9**

θ	n	ML estimation			Moment estimation		
		$\hat{\theta}$	Var ($\hat{\theta}$)	MSE ($\hat{\theta}$)	$\tilde{\theta}$	Var ($\tilde{\theta}$)	MSE ($\tilde{\theta}$)
0.2	25	**0.376**	0.000114	0.083	**0.121**	0.006	0.029
	50	**0.305**	0.000025	0.083	**0.123**	0.002	0.024
	100	**0.274**	0.000009	0.083	**0.178**	0.022	0.035
0.5	25	**0.759**	0.000251	0.119	**0.389**	0.021	0.020
	50	**0.621**	0.000115	0.121	**0.421**	0.015	0.011
	100	**0.562**	0.000056	0.107	**0.491**	0.019	0.043
0.8	25	**1.404**	0.000035	0.922	**0.654**	0.023	0.042
	50	**0.952**	0.000018	0.922	**0.742**	0.008	0.034
	100	**0.846**	0.000009	0.916	**0.778**	0.005	0.019
1.0	25	**1.201**	0.000085	0.511	**0.848**	0.019	0.086
	50	**1.080**	0.000023	0.462	**0.896**	0.015	0.027
	100	**1.019**	0.000008	0.332	**0.972**	0.007	0.018
1.3	25	**1.498**	0.000094	0.265	**1.023**	0.046	0.034
	50	**1.414**	0.000015	0.265	**1.129**	0.016	0.012
	100	**1.346**	0.000008	0.246	**1.241**	0.006	0.008
1.6	25	**1.711**	0.000078	0.203	**1.358**	0.019	0.042
	50	**1.701**	0.000052	0.202	**1.491**	0.008	0.015
	100	**1.642**	0.000019	0.201	**1.538**	0.001	0.009
2.0	25	**2.245**	0.000077	0.195	**1.718**	0.014	0.055
	50	**2.202**	0.000059	0.192	**1.856**	0.009	0.023
	100	**2.105**	0.000009	0.192	**1.941**	0.003	0.012
2.4	25	**2.556**	0.000085	0.165	**2.246**	0.019	0.072
	50	**2.522**	0.000047	0.154	**2.317**	0.009	0.067
	100	**2.443**	0.000018	0.143	**2.378**	0.009	0.023
3.0	25	**3.108**	0.000051	0.116	**2.765**	0.024	0.085
	50	**3.091**	0.000015	0.112	**2.808**	0.014	0.056
	100	**3.044**	0.000009	0.112	**2.953**	0.007	0.028