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The Best Common Feature Between ii-Open and α −Open Sets

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Abstract:

It is well known that for any topological space the family of all α −open sets is also a topological space. In this paper, we introduce a new description of ii-open sets in a topological space with proof that the family of ii-open sets is again a topological space.

Keywords:

α −open sets and ii-open sets

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1. Introduction and Preliminaries

A systematic study of ii-open sets in topological spaces are presented recently in [8], [4] and [5]. The semi-open [6] and α −open sets [9] are considered the start of the appearance of the new classes of open sets that they came after, see for example [1], [3], [8] and [2].

Let (X, τ) be a topological space. Recall that, $M \subseteq X$ is open, if $M \in \tau$ or for all $m \in M$ there exists $O \in \tau$ such that $m \in O \subseteq M$. Also, M is closed if $M^C$is open, M is α −open if $M \subseteq int(cl(int(M)))$, M is int-open [8] if there exists $\phi, X \neq O \in \tau$ such that $int(M) = O$ and M is i-open [7] if there exists $\phi, X \neq O \in \tau$ such that $M \subseteq cl(M \cap O)$.

In this paper, we denoted by τ°, τ^C, τ^i, $\tau^{int}$and τ^α to be the family of open, closed, i-open, int-open and α −open sets respectively. Finally, $M \subseteq X$ is said to be to be ii-open if $M \in \tau^i \cap \tau^{int}$ and we denote by τ^{ii} to be the family of all ii-open sets.

2. Proving (X, τ^{ii}) is a topological space.

We begin this section by the following theorem:

Theorem2.1. ([7], [8]) For any topological space, all open sets are int-open, i-open, ii-open and α −open. Further, all α −open and ii-open sets are i-open.

Note that the converse of above theorem is not true in general.

Example2.2. Let $X = \{0,1,2, \dots \dots . .\}$,

$\tau = \{\phi, \{0,2,3, \dots \dots . .\}, \{1,2,3, \dots \dots\}, \{2,3,4, \dots \dots\}, \{0\}, X\}$,

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$M=\{0,2\}$ is ii-open not $\alpha-$open because $int(M)=\{0\}$, so $M\in\tau^{int}$.

Now, $M\subseteq Cl(M\cap\{0,2,3,\dots\dots\})$, thus, $M\in\tau^{i}$ and therefore, $M\in\tau^{ii}$.

But $Cl(int(M))=\{0\}$ and $M=\{0,2\}\nsubseteq int(Cl(int(M)))$.

Example2.3. Let $X=\{0,1,2\}$and $\tau_1=\{\phi,\{0\},X\}$, $\tau_2=\{\phi,\{0\},\{1,2\},X\}$, $\tau_3=\{\phi,\{0\},\{1\},\{0,1\},\{0,2\},X\}$. Then, $\tau_1^{\alpha}=\tau_1^{int}=\tau_1^{i}=\tau_1^{ii}=\{\phi,\{0\},\{0,1\},\{0,2\},X\}$ and $\tau_2^{int}=\{\phi,\{0\},\{1,2\},\{0,1\},\{0,2\},X\}$, $\tau_2^{i}=\{\phi,\{0\},\{1,2\},\{1\},\{2\},X\}$. Therefore, $\tau_2^{ii}=\tau_2^{i}\cap\tau_2^{int}=\{\phi,\{0\},\{1,2\},X\}$ and$\tau_2=\tau_2^{\alpha}=\tau_2^{ii}$. Finally, $\tau_3^{int}=\{\phi,\{0\},\{1\},\{0,1\},\{0,2\},\{1,2\},X\}$, $\tau_3^{i}=\{\phi,\{0\},\{1\},\{0,1\},\{0,2\},\{2\},X\}$. Therefore, $\tau_3^{ii}=\tau_3$and $\tau_3^{\alpha}=\tau_3^{ii}$.

Now, from Theorem 2.1 and above examples we conclude the following diagram:

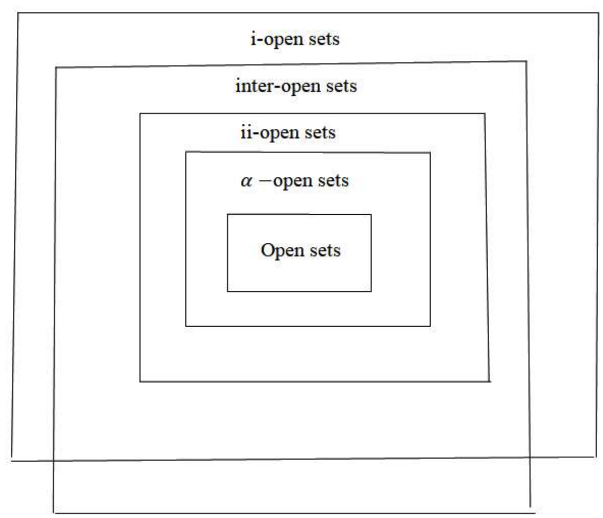

Fig.1.

Now, we prove our main result:

Theorem2.4. *(X, τ^{ii})* is a topological space.

Proof. Let $M \subseteq X$ be ii-open set. We will prove that M is open in *(X, τ^{ii})*. That is, for all $m \in M$ there exists $O^* \in \tau^{ii}$ such that $m \in O^* \subseteq M$. Since M is ii-open set, it follows that there exists $\phi, X \neq O, K \in \tau$ such that $int(M) = O$ and $M \subseteq Cl(M \cap K)$. Now, for $m \in M$ we have the following two cases:

(i) $m \in O$.
(ii) $m \notin O$.

Case (i) $m \in O$**.** By Theorem 2.1, O is ii-open, so put $O = O^*$ we have, $m \in O^* = O = int(M) \subseteq M$. Thus, M is open in τ^{ii}. Case (ii) $m \notin O$**.** Consider the following sets, $O_\circ = \{m\} \cup O$ and $O_n = \{m\} \cup \{m_i\}_{i=1}^{n} \cup O$, $\forall n = 1,2,..$ with $m_i \in M$ $\forall i$. Put $O^* = O_n$ for some $n \in \{0,1,2, \dots \dots . .\}$. Therefore, it is enough to see that O^* is ii-open. Clearly, $O^* \subseteq Cl(O^* \cap O)$ and $int(O^*) = O$. This implies, $m \in O^* \subseteq M$. Now, since every set in τ^{ii} is open, it follows that τ^{ii} is closed under arbitrary union and finite intersection. This complete the proof.

Remark2.5. Let $M \subseteq X$**,** then M is ii-open if $M \subseteq Cl(M \cap O)$ and $int(M) = K$. If we take $O = K$, then every α −open set is ii-open. Indeed, let M be α −open. If $int(M) = \phi$, then there is nothing to proof. So, $int(M) \neq \phi$ say $int(M) = K$, that is $M \in \tau^{int}$. Now, since M is α −open we have, $M \subseteq int(Cl(int(M)))$, and therefore, $M \subseteq Cl(int(M)) \subseteq Cl(M \cap K)$. Thus, $M \in \tau^{i}$. That is, $M \in \tau^{ii}$. Note that, the choosing $O \neq K$ in the definition of ii-open sets has no effect as shown in following example:

Let $X = \{0,1,2,3\}$ and $\tau = \{\phi, \{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3\}, X\}$.

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$\tau^{int} = \{\phi, \{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3\}, \{0,2\}, \{0,3\}, \{0,2,3\}, X\}.$

$\tau^{i} = \{\phi, \{0\}, \{0,1\}, \{0,1,2\}, \{0,1,3\}, \{1\}, \{2\}, \{3\}, \{0,2\}, \{0,3\}, \{1,2\},$

$\{1,3\}, \{0,2,3\}, \{1,2,3\}, X\}.$

$\tau^{\alpha} = \tau^{int} \neq \tau, \tau^{ii} = \tau^{int}.$

Conclusions

From above we concluded that ii-open set is a generalization of α −open set and it has the same property that the family of ii-open sets is always a topological space and there is no relation among the inter-open set and i-open set, while all ii-open sets are i-open and int-open.

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